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Prime Cordial Labeling of Some Graphs Related to H-Graph

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Abstract. A prime cordial labeling of a graph G with vertex set V(G) is a bijection f : V(G) → $\{1, 2, 3, \ldots, |V(G)|\}$ such that each edge uv is assigned the label 1 if gcd(f(u), f(v)) = 1and 0 if gcd(f(u), f(v)) > 1, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits prime cordial labeling is called prime cordial graph. In this paper, we prove that the graphs $H\Theta K_1$, P(r.H), C(r.H) and S(r.H) are prime cordial.

Keywords: Prime cordial labeling, H-graph, path union, cycle union and open star of graphs.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

We consider only simple, finite, undirected and non-trivial graph G = (V(G), E(G)) with the vertex set V(G) and the edge set E(G). The number of elements of V(G), denoted as |V(G)| while the number of elements of E(G), denoted as |E(G)|. For standard terminology and notations we follow Harary [3]. A graph labeling is an assignment of labels to edges, vertices or both. Cahit. I [1] introduced the concept of cordial labeling in 1987. The concept of cordial labeling was extended to divisor cordial labeling, sum divisor cordial labeling, prime cordial labeling, total cordial labeling, etc., A survey of graph labeling, we refer to Gallian [2].

Vaidya and Shah [10] proved that, some star and bistar related graphs are divisor cordial graphs. Duplication of vertices and edges was introduced by Vaidya and Barasara [9] and they applying this concept to the product cordial graphs. Sugumaran and Rajesh [5] have shown that, Swastik graph S_n, some graph operations related to Swastik graph, Jelly fish J(n, n) and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh [6] proved that Theta graphs and some operations of Theta graph are sum divisor cordial graphs. Sundaram et al. [8] introduced the concept of prime cordial labeling. Sugumaran and Prakash [7] proved that one point union of path of Theta graphs, open star of Theta graphs and path union of even copies of Theta graph are prime cordial graphs.

Sugumaran and Mohan [4] have proved prime cordial labeling of the graphs such as butterfly graph, W-graph, H-graph and duplication of edges of an H-graph. In section 2, we summarize the necessary definitions and notations which are useful for the present work. In section 3, we proved that the graphs such as $H\Theta K_1$, path union of r copies of H-graph, cycle union of r copies of H-graph and open star of r copies of H-graph are prime cordial graphs. An application of graph labeling is discussed in [11].

2. Definitions

In this section, we will provide a brief summary of definitions, which are necessary for the present investigation.

Definition 2.1. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called *binary vertex labeling* of G and f(v) is called the label of the vertex v of G under f.

Definition 2.2. A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where

$v_f(i)$ = number of vertices of G having label i

$e_f(i)$ = number of edges of G having label i

Definition 2.3. [8]A *prime cordial labeling* of G with vertex set V(G) is a bijection $f : V(G) \rightarrow \{1, 2, 3, \ldots, |V(G)|\}$ such that each edge uv is assigned the label 1 if gcd (f(u), f(v)) = 1 and 0 if gcd (f(u), f(v)) > 1, then the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1. A graph which admits prime cordial labeling is called prime cordial graph.

Definition 2.4. The *graph $H\Theta K_1$* is obtained by adding a pendant edge to each vertex of an H-graph.

Definition 2.5. The *path union of a graph* G is the graph obtained from a path P_n $(n\geq 2)$ by replacing each vertex of the path by graph G and it is denoted by P(n.G).

Definition 2.6. The *cycle union of a graph* G is the graph obtained from a cycle C_n $(n\geq 3)$ by replacing each vertex of the cycle by graph G and it is denoted by C(n.G).

Definition 2.7. The *open star of a graph* G is the graph obtained from a star graph $K_{1, n}$ $(n\geq 2)$ by replacing each vertex(except the apex vertex) of the star by graph G and it is denoted by S(n.G).

3. Main results

In this section, we proved that some of the graphs related to H-graph are prime cordial graphs.

Theorem 3. 1. The graph $H\Theta K_1$ admits prime cordial labeling.

Proof: Consider an H graph with 2n vertices. Let G = HʘK$_1$. Let V(H) = $\{u_i, v_i : 1 \leq i \leq n\}$. Let $u_1', u_2', u_3', \dots, u_n'$, be the pendant vertices connected to $u_1, u_2, u_3, , \dots, u_n$ respectively and let $v_1', v_2', v_3', \dots, v_n'$, be the pendant vertices connected to $v_1, v_2, v_3, \dots, v_n$ respectively in G. Then |V(G)| = 4n and |E(G)| = 4n – 1. We define the vertex labeling function f : V(G) → $\{1, 2, 3, \dots, 4n\}$ as follows.
f (u_i) = 4i – 3 ; $1 \leq i \leq n$.
f (v_i) = 4i – 2 ; $1 \leq i \leq n$.
f (u_i') = 4i – 1 ; $1 \leq i \leq n$.
f (v_i') = 4i ; $1 \leq i \leq n$. In view of the labeling pattern defined above, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a prime cordial graph.

Example 3. 1. Prime cordial labeling of the graph H$_5$ʘK$_1$ is shown in Figure 1.

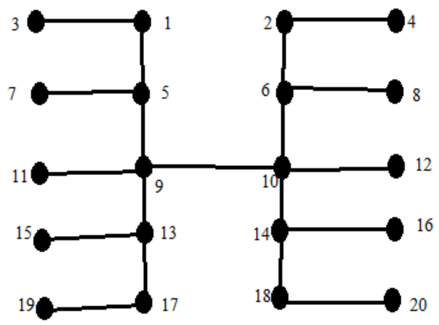

Figure 1: Prime cordial labeling of H$_5$ʘK$_1$

Theorem 3.2. The Path union of r copies of H-graph is a prime cordial graph.
Proof: Consider an H-graph with 2n vertices. Let G = P(r.H) be the Path union of r copies of H-graph. In graph G, |V(G)| = 2nr and |E(G)| = 2nr – 1. We denote u_i^k and v_i^k are the i^{th} vertex in the k^{th} copy of the first and second path in the H-graph respectively, where i = 1, 2, 3, … , n and k = 1, 2, 3, … , r. Notice that the vertices v_1^k and v_1^{k+1} are connected by an edge in G, where k = 1, 2, 3, … , r – 1. To define the vertex labeling function f : V(G) → $\{1, 2, 3, \dots, 2nr\}$ as follows.
f (u_i^k) = 2i – 1 ; (k – 1)n + 1 ≤ $i \leq kn$, k = 1, 2, 3, … , r.
f (v_i^k) = 2i ; (k – 1)n + 1 ≤ $i \leq kn$, k = 1, 2, 3, … , r.
If n is even, then we interchange the labels of the vertices $v_{\frac{n}{2}}^k$ with $v_{\frac{n}{2}+1}^k$, k = 1, 2, 3, … , r. In view of the labeling pattern defined above, we have $|e_f(0) - e_f(1)| = 1$. Hence G is a prime cordial graph.

Example 3. 2. Prime cordial labeling of the graph P(3. H$_4$) is shown in Figure 2.

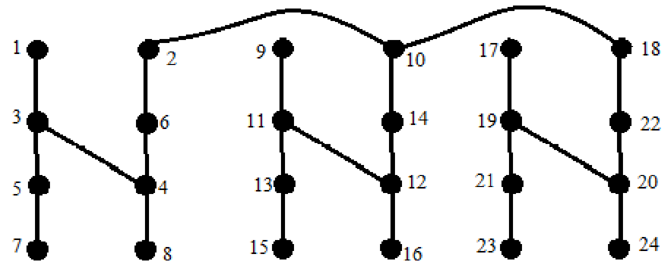

Figure 2: Prime cordial labeling of graph P(3. H_4)

Theorem 3.3. The Cycle union of r copies of an H-graph is a prime cordial graph.

Proof: Consider an H-graph with 2n vertices. Let G = C(r.H) be the cycle union of r copies of H-graph. In graph G, |V(G)| = 2nr and |E(G)| = 2nr. We denote u_i^k and v_i^k are the i^{th} vertex in the k^{th} copy of the first and second path in the H-graph respectively, where i = 1, 2, 3, … , n and k = 1, 2, 3, … , r. Notice that the vertices v_1^k and v_1^{k+1} are connected by an edge and the vertices v_1^r and v_1^1 are connected by an edge in G, where k = 1, 2, 3, … , r – 1. To define the vertex labeling function f : V(G) → $\{1, 2, 3, \ldots, 2nr\}$ as follows.

f (u_i^k) = 2i – 1 ; (k – 1)n + 1 ≤ $i \ \leq kn$, k = 1, 2, 3, … , r.

f (v_i^k) = 2i ; (k – 1)n + 1 ≤ $i \ \leq kn$, k = 1, 2, 3, … , r.

If n is even, then we interchange the labels of the vertices $v_{\frac{n}{2}}^k$ with $v_{\frac{n}{2}+1}^k$, k = 1, 2, 3, … , r. In view of the labeling pattern defined above, we have |e_f (0) - e_f (1)| = 0. Hence G is a prime cordial graph.

Example 3.3. Prime cordial labeling of the graph C(4. H_3) is shown in Figure 3.

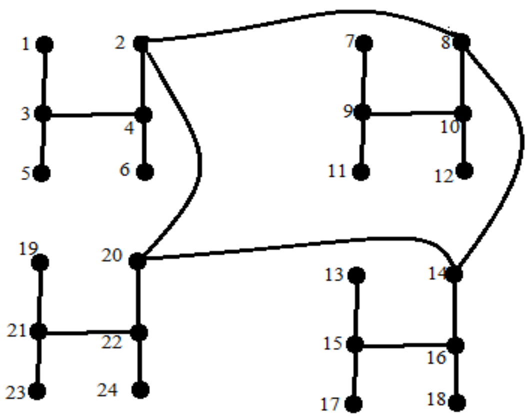

Figure 3: Prime cordial labeling of C(4. H_3)

Theorem 3.4. The Open star of r copies of an H-graph is a prime cordial graph.

Proof: Consider an H-graph with 2n vertices. Let G = S(r.H) be the open star of r copies of H-graph. In graph G, |V(G)| = 2nr + 1 and |E(G)| = 2nr. We denote u_i^k and v_i^k are the i^{th} vertex in the k^{th} copy of the first and second path of the H-graph respectively, where i = 1, 2, 3, … , n and k = 1, 2, 3, … , r. Let w be the apex vertex of G. Also we join the vertices v_1^k with w by an edge in G, where k = 1, 2, 3, … , r. We define the vertex labeling function f : V(G) → $\{1, 2, 3, \ldots, 2nr + 1\}$ as follows.

f (u_i^k) = 2i – 1 ; (k – 1)n + 1 ≤ $i \le kn$, k = 1, 2, 3, … , r.

f (v_i^k) = 2i ; (k – 1)n + 1 ≤ $i \le kn$, k = 1, 2, 3, … , r.

f (w) = 2nr + 1.

If n is even, we interchange the labels of the vertices $v_{\frac{n}{2}}^k$ with $v_{\frac{n}{2}+1}^k$, k = 1, 2, 3, … , r and further we interchange the labels of the vertices w with v_n^r. In view of the labeling pattern defined above, we have $|e_f(0) - e_f(1)| = 1$. Hence G is a prime cordial graph.

Example 3. 4. Prime cordial labeling of the graph S(4. H_3) is shown in Figure 4.

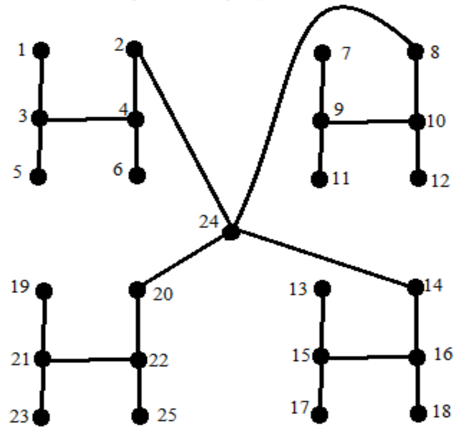

Figure 4: Prime cordial labeling of S(4. H_3)

4. Conclusion

H-graph is one of the interesting graphs in graph theory. In this paper we proved that the graphs such as H⊙K_1, path union of r copies of H-graph, cycle union of r copies of H-graph and open star of r copies of H-grah are prime cordial graphs. Extending our results to various other graph operations related to H-graph is an interesting open area of research.

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