Annals of Pure and Applied Mathematics
Vol. 16, No. 1, 2018, 215-222
ISSN: 2279-087X (P), 2279-0888(online)
Published on 15 January 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v16n1a23

Structures of Anti-Inverse Semirings

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Received 10 September 2017; accepted 26 November 2017

Abstract. In this paper we study the structures of an anti-inverse semiring. It has been proved that, for a semiring (S, +,·), if (S,·) is anti-inverse idempotent semigroup, we define a relation 'σ' on a semigroup S by a σ b implies $ab^n = b^{n+1}$, $ba^n = a^{n+1}$ for any positive integer n and for any *a, b* in S then σ is congruence on S and also S is distributive.we proved that if (S,+,.) be an anti-inverse semiring then S is Quasi-separative, weakly separative, separative and (S,.) is normal. If (S ,.) be an anti-inverse Archimedean semigroup and if S is weakly separative then it is weakly reductive. If (S,+,.) be an anti-inverse semiring, we define a relation ρ on a semigroup S as *a ρ b* if and only if $a^2 = ab = ba$ for all *a, b* in S then (S,+,., ρ) is a partially order semiring. We determine the additive and multiplicative structure of these anti-inverse semirings and the modern interest in semirings arises primarily from fields of applied mathematics such as optimization theory, the theory of discrete event dynamical systems, automata theory, as well as from the allied areas of theoretical computer science and theoretical physics.

Keywords: Congruence, Permutable, Separative, Partial order semirings.

AMS Mathematics Subject Classification (2010): **16Y60**

1. Introduction

Semirings is an algebraic structure, similar to a ring but without the requirement that each element must have a multiplicative identity or an additive zero. Depending on how much other ring – like properties are also cancelling or added, various different concepts of semirings (S,+,.) have been considered in the literature, since in 1934 the first abstract concept of semiring was introduced by Vandiver[17]. Vasanthi and Amala[18] proved results on some special classes of semirings and ordered semirings. Chowdary et al. [3,4], Rajeswari [10] were proved some results in structural properties of semirings and on invertibility matrices over semirings. Sharp [12] was first introduced anti-inverse elements in a semigroup. In 1982, The anti-inverse semigroups were studied by Blagojevic [3]. The first step in the Archimedean semigroups has been made by Tamura [16]. The research of separative semigroup was being began from the famous paper of

Hoeitt and Zuclevman [9]. Drazin [4] introduced the term 'quasi-separativity' and studied connection between it and other semigroup properties. They proved some results on commutative separative semigroup. Venkateswarlu et al. [19] studied "Boolean Like Semirings", Nagy [1], Pondelicek [2] proved the least separative, separative congruence on a weakly commutative semigroup. Ghosh [5] studied on the class of idempotent semirings and also we follow the terminology proposed by Shobhalatha and Bhat [14,15] in an anti-inverse semirings. Heinz Mitsch [6] defined partial order relation on a semigroup.

1.1. Main results

In contrast to the above said references , if (S,+,.) is an idempotent semiring in which (S,.) is defined by $a \circ b = aab$ for all a ,b in S then (S,+,.) is a Boolean semiring and it has been proved some additive and multiplicative structures of an anti-inverse semiring as noted in abstract.

2. Preliminaries

Definition 2.1. A semiring is a nonempty set S on which operations of addition '+'and multiplications '.' have been defined such that the following are satisfied:

i) (S,+) is a semigroup,
ii) (S,.) is a semigroup,
iii) Multiplication distributes over addition from either side.

Examples of semiring

Example 2.1(a). The set of natural numbers under the usual addition, multiplication,
Example 2.1(b). Any ring (R,+,.)

Definition 2.2. A semigroup S is called anti-inverse if for each element 'a' in S there is an element 'b' in S such that $aba = b$ and $bab = a$. The elements a and b are then called anti-inverses.

Example 2.2(a).

$\cdot$	a	b
a	a	b
b	b	a

a and b are their own anti-inverses since $aaa = a$, $bbb = ab = b$.
$aba = b$, $bab = a$, a and b are anti-inverses.

Definition 2.3. An element 'x' in a semigroup (S,+, .) is said to be idempotent if $x.x = x$ *and* $x+x = x$.

Definition 2.4. A semigroup (S,.) is called left (right) permutable if forevery a,b,c in S, $abc = acb$ $(abc = bac)$ and is permutable if it is both left and right permutable.

Definition 2.5. A semigroup (S,.) is called quasi-separative if $x^2 = xy = yx = y^2$.

Definition 2.6. A semigroup (S,.) is said to be weakly separative if $x^2 = xy = y^2 \Rightarrow x = y$ for all x,y in S.

Definition 2.7. A semigroup (S,.) is called quasi commutative if for any a,b in S we have $ab = b^r a$ for some positive integer r.

Definition 2.8. A semigroup (S,+,.) is completely regular if there exist x in S such that $a= a+x+a$, $a+x = x +a$ and if $a(a+ x) = a+ x$ then S is quasi-completely regular. A semiring S is said to be completely regular if for every element a of S is completely regular.

Definition 2.9. A system (S,+,.) a Boolean semiring if and only if the following properties hold:

i) (S,+) is an abelian group.
ii) (S,.) is a semigroup.
iii) $a(b+c) = ab + ac$ and
iv) $abc = bac$, for all a,b,c in S.

3. Some structural properties of semirings

Lemma 3.1. Let (S, .) be an anti-inverse semigroup then $ab^na = b^n$, for all a, b in S and n is any positive integer.

Proof: We prove it by the method of Mathematical Induction.

Let $P(n)$: $ab^na = b^n$, for all a,b in S

$P(1)$: $aba = b \Rightarrow P(1)$ is true.

Assume $n = k$ such that $ab^ka = b^k$

Consider $ab^{k+1}a = ab^kba = a(ab^ka)(aba)a = a.a.\ b^k.a.a.b.a.a = aa(ab^ka)(aba)$

$= (aaa)b^kab = (ab^ka)b = b^k.b = b^{k+1.}$

Thus by the principle of Mathematical Induction $P(n)$ is true for all n in N.

Theorem 3.2. let (S,·) be an Idempotent anti-inverse semiring we define a relation 'σ' on a semigroup S by $a\ \sigma\ b$ implies $ab^n = b^{n+1}$, $ba^n = a^{n+1}$ for any positive integer n and for any a , b in S then σ is a congruence on S.

Proof: It has to be first prove that σ is an equivalence relation on S.

Since $a.a^n = a^{1+n} = a^{n+1} \Rightarrow a\ \sigma\ a \Rightarrow \sigma$ is reflexive.

Suppose $a\ \sigma\ b \Rightarrow ab^n = b^{n+1}$ …….(1) and $ba^n = a^{n+1}$…….(2)

Replace a by b in (1) and b by a in (2), we get $ba^n = a^{n+1}$, $ab^n = b^{n+1}$ .Thus σ is symmetric.

Suppose $a\ \sigma\ b \Rightarrow ab^n = b^{n+1}$ and $ba^n = a^{n+1}$

$b\ \sigma\ c \Rightarrow bc^m = c^{m+1}$ and $cb^m = b^{m+1}$ for all m,n in Z^+

$ab^n = b^{n+1} \Rightarrow ab^n c^m = b^n.b.c^m \Rightarrow ac^{m+n} = b^n c^{m+1} = c^{m+1+n} = c^{m+n+1.}$

Similarly we can prove that $ca^{m+n} = a^{m+n+1} \Rightarrow a\ \sigma\ c$. Thus σ is transitive.

Therefore σ is an equivalence relation.

To prove compatibility,

Consider $(ac)(bc)^n = acb^n.c^n = a(a.c.a)b^n.c^n = acab^n.c^n = cb^n.c^n = (bcb)(cb^nc)c^n$

$= b(cbc)b^n.c.c^n = b.b.\ b^n.c.c^n = .b.\ b^n.c.c^n = b^{n+1}.c^{n+1} = (bc)^{n+1} \Rightarrow ac\ \sigma\ bc$

And $(bc)(ac)^n = bca^n.c^n = b(b.c.b)a^n.c^n = (bb)cba^nc^n = b.c.b.\ a^n.c^n = c.\ a^n.c^n$

$= (a.c.a)(c.a^n.c).c^n = a.a.\ a^n.c.c^n = a.a^n.c.c^n = a^{n+1}.c^{n+1} = (ac)^{n+1} \Rightarrow bc\ \sigma\ ac.$

Therefore σ is a congruence on S.

Theorem 3.3. Let (S,+, .) be an anti-inverse semiring then (S,+, .) i) Quasi-separative, ii) Weakly separative, iii) Separative.

Proof: Since S is anti-inverse, for some a,b,c in S then $a + b + a = b$ and $aba = b$.
Consider $a + a = a + b \Rightarrow a + a + a = a + b + a \Rightarrow a = b$, also consider $a + b = b + b \Rightarrow b + a + b = b + b + b \Rightarrow a = b$. Thus $a + a = a + b = b + a \Rightarrow a = b$.
Hence (S,+) is quasi separative.
Consider $a^2 = a.a = a.b \Rightarrow a.a.a = a.b.a \Rightarrow a = b$ and also consider $ab = b^2 \Rightarrow ab = b.b \Rightarrow bab = b.b.b \Rightarrow a = b$.Therefore $a^2 = a.b = b^2 \Rightarrow a = b$.
=> (S, +, .) is separative (1)
Also consider $a + a = a + b \Rightarrow a = b$ and $b + a = b + b \Rightarrow a = b$.
We need to prove that $a + b = b + a \Rightarrow a = b$. Consider $a+b = b+a \Rightarrow a+ (a+b) = a+(b+a) \Rightarrow (a+a)+b = b \Rightarrow a+b+b = b$ [Since $a + a = a + b$]
$\Rightarrow b + a + b + b + b = b + b + b \Rightarrow b + a +(b+b+b) = b \Rightarrow b+a+b = b \Rightarrow a = b$.
Therefore $a + a = a +b = b + a = b + b \Rightarrow a = b$.
Similarly we have $a^2 = ab = ba =b^2 \Rightarrow a = b \Rightarrow$ (S,+,.) is weakly separative (2)
From (1) and (2), (S, +, .) is separative.

Theorem 3.4. let (S,+, .) be an anti-inverse semiring then (S,.) is left normal, right normal and normal.

Proof: Since (S,.) is an anti-inverse, for some a,b,c in S, $aba =b$.
Consider $abc = a(cbc)(bcb) = ac(bcb)cb = ac.c.cb = acb \Rightarrow$ (S, .) is left normal.
Also $bca = (cbc)(bcb)a = c(bcb)cba = c.c.c.ba = cba \Rightarrow$ (S,.) is right normal.
Now consider $abca = a(cbc)(bcb)a = ac(bcb)cba = ac.c.c.ba = acba \Rightarrow$ (S,.) is normal.

Lemma 3.5. For any a,b in S,$aba = bab$.

Proof: $aba = (bab)(aba)(bab) = b(aba)(bab)ab = b.b.a.a.b = bab$.

Theorem 3.6. If (S,+,.) be an idempotent semiring in which (S,.) is defined by $a\circ b = aab$ for all a,b in S then (S,+,.) is a Boolean semiring.

Proof: Since $a\circ a = a.a.a = a$, $a\circ 0 = 0\circ a = 0$ for all a in S.
For a,b,c in S
$a\circ b = aab = (bab)(bab)(aba) = ba(bb)a(bab)a = bab.a.a.a = b(aba)a = bbaa = bba$
$= b\circ a$.
$a\circ(b\circ c) = a\circ(bbc) = aabbc = a(bab)bbc = (aba)(bbb)c = ab(aa)(bb)bc = (aba)abbc$
$= bab.bc = abc = a\circ(b\circ c)$.
Therefore (S, ∘) is a commutative semigroup.
Consider $a\circ(b + c) = aa(b+ c) = aab + aac = a\circ b + a\circ c$
and $(a + b) \circ c = c\circ(a + b)$ [since (S, ∘) is commutative] $= c\circ a + c\circ b = a\circ c + b\circ c$
Thus $a\circ(b +c) = (a +b) \circ c$
Since $a + a = 0$ for all a in S, every element of S has additive inverse.
Also $a\circ b\circ c= (aab)\circ c=(aab)(aab)c = (bab)a(aba)(bab)(bab)(aba)c$
$= b(aba)a(bab)abb(aba)bac = b.b.a.a.a.b.b.b.b.a.c = (bba)(bba)c$
$= (bba) \circ c = b\circ a \circ c$.Hence (S,+, ∘) is a Boolean semiring.

Theorem 3.7. Let (S,.) be an anti-inverse Archimedean semigroup. If S is weakly separative then it is weakly reductive.

Proof: Let S be an Archimedean anti-inverse semigroup. Assume that S is weakly separative. $x^2 = xy = y^2 \Rightarrow x = y$ for all x,y in S.To prove that S is weakly reductive, we need to prove for any a in S, if $ax = ay$, $xa = ya \Rightarrow x = y$.

Since S is Archimedean, for x, y in S and there exists some positive integers m, n such that $x^m = uav$ and $y^n = zaw$ for some u, v, z, w in S. Where S′ = S U {1}

Let $ax = ay$, $xa = ya$. Consider, $x^{m+1} = x^m.x$

$x^{m+1} = (uav)\, x = (aua)a(ava)x = (au)(aaa)(vax) = (au)a(vax) = (aua)v(ax) = u.v.ay$ [Since $ax = ay$] $x^{m+1} = u(ava)a(aya) = uav(aaa)ya = uavaya = uav(aya) = (uav)y = x^m y$ (1)

and $y^{n+1} = y^n.y = (zaw)y = z(awy) = (aza)a(awa)y = (az)(aaa)(way) = (aza)w(ay)$

$= zw(ax) = z(awa)a(axa) = zaw(aaa)xa = zaw(axa) = (zaw)x = y^n.x$ (2)

For $m \geq 2$, $(x^m)^2 = x^{2m} = x^{2m-2}.x^2 = x^{2m-2}.xy = x^{m-2}.x^m.xy = x^{m-2}.x^{m+1}.y = x^{2m-2}.y^2 = (x^{m-1}.y)^2$ $\Rightarrow x^m = x^{m-1}.y$

After $(m-1)$ steps $x^2 = xy$ (3)

Similarly for $n \geq 2$, $(y^n)^2 = y^2.y^{2n-2} = y.y^{n+1}.y^{n-2} = y.y^n.x.y^{n-2} = y.(zaw)xy^{n-2} = y(aza)a.w.a.x.y^{n-2} = (ya)z.a.wa.xy^{n-2} = (xa)zwxy^{n-2} = xa(xzx)wxy^{n-2} = a.z.w.y^{n-2} = (xax)(aza)wy^{n-2} = x.x.zawy^{n-2} = x^2.zawy^{n-2} = x^2.y^n.y^{n-2} = x^2.y^{2n-2} = (xy^{n-1})^2 \Rightarrow y^n = xy^{n-1}$.

After $(n-1)$ steps $y^2 = xy$ (4)

From (3) and (4), $x^2 = xy = y^2 \Rightarrow x = y$. [Since S is weakly reductive]

Hence (S, .) is weakly reductive.

Theorem 3.8. If (S,.) is an idempotent anti-inverse semigroup then (S,.) is σ – reflexive, permutable, quasi-commutative.

Proof: To prove that (S,.) is σ – reflexive, we apply the method of Mathematical Induction. let $P(n) : ab = (ba)^n$.

We have $aba = b \Rightarrow abab = bb \Rightarrow aba(aba) = ba \Rightarrow ab(aa)ba = ba \Rightarrow ab(aba) = ba$ $\Rightarrow abb = ba \Rightarrow ab = ba \Rightarrow P(1)$ is true.

Assume that $P(k)$ is true that is $ab = (ba)^k$, for some positive integer k.

To prove $P(k+1)$ is true, We have $ab = (ab)^k.ab.(ab)^k$ for some positive integer k.

$ab = ab.ab.(ab)^k = a(a.b)b(ab)^k = ab(ab)^k = (ab)^{k+1} \Rightarrow P(k+1)$ is true.

Hence $P(n)$ is true for all n in N.Therefore (S,.) is σ – reflexive.

To prove (S,.) is Permutable,

consider $abc = a(cbc)(bcb) = ac(bcb)cb = acccb = acb$ (1)

Since S is anti-inverse,

we have $bab = a \Rightarrow ba(bcb) = acb \Rightarrow bac = acb \Rightarrow acb = bac$ (2)

From (1) and (2) , $abc = acb = bac$. Thus (S,.) is permutable.

To prove (S,.) is quasi-commutative, let us consider $P(n)$: $ab = b^k a$ for some k in z+

$aba = b \Rightarrow aba.ba = b.ba \Rightarrow ab.(aba) = (bb)a \Rightarrow abb = ba \Rightarrow ab = ba \Rightarrow P(1)$ is true.

$ab = b^k(ab)b^k = ab.b.b^k = abb^k = ab^{k+1} \Rightarrow P(k+1)$ is true.

Therefore $P(n)$ is true for all n in N.

Thus $ab = (ba)^n$ is true. Hence (S,.) is quasi-commutative.

Theorem 3.9. If (S,.) is an anti-inverse idempotent quasi-commutative semigroup then (S,.) is weakly commutative.

Proof: It has to be prove that $(ab)^k = xa = by$ for some x,y in S and k is any positive integer.Consider $(ab)^k = a^k.b^k = xa^kx.b^k = x.xa(ab^ka) = xx.aa.ab = xxab = xa(aba)$
$= xa(bab)ba = xababa = xaaa = xa.$
And also consider $(ab)^k = a^k.b^k = ba^kb.yb^ka = b.(ba)y(ab) = b.b.y.b = by.$
Thus $(ab)^k = xa = by.$ => (S, .) is weakly commutative.

Theorem 3.10. If (S,+,.) is an idempotent anti-inverse semiring then (S,+,.) is completely regular and hence quasi completely regular.
Proof: Since (S,.) is an anti-inverse semiring, we have $axa = x.$
$(xax)(axa)a = axa \Rightarrow x(axa)xa.a = axa$
Post multiplied by 'a' $x.x.x.a.a.a. = axa.a \Rightarrow x.a = a.axa \Rightarrow xa = ax$ and
$xax = (axa)(xax)x = a(xax)ax.x = aaa.x.axa = a.xaxa = ax(aa)xa = (axa)(axa) = x.x = x.$
Since (S,+) is an anti-inverse semiring, we have $a+x+a = x \Rightarrow a+a+x+a = a+x$
$\Rightarrow x+a+x+a+x+a = a+x \Rightarrow x+x+x+a = a+x \Rightarrow x+a = a+x.$
And also $x+a+x = (a+x=a)(x+a+x)+x = a+(x+a+x)+a+x+x = a+a+a+x+a+x+a$
$= a+x+a+x+a = a+x+(a+a)+x+a = x+x = x$.Hence (S,+,.) is completely regular.
$a(a+x) = a.a(a+x) = a.a.a + a.a.x = a + axa$ [since $xa = ax$] $\Rightarrow a(a+x) = a+x.$
Hence (S,+,.) is quasi-completely regular.

Theorem 3.11. Let (S,+,.) be an anti-inverse semiring. Define a relation ρ on a semigroup S as follows $a \rho b$ if and only if $a^2 = ab = ba$ for all a,b in S then (S,+,., ρ) is a partially order semiring.
Proof: Define a relation ρ on a semigroup 'S' as follows.
$a \rho b$ if and only if $a^2 = ab = ba$ for all a,b in S.
We have $a^2 = a.a = a.a = a \rho a$.Therefore ρ is reflexive.
Let $a \rho b$ and $b \rho a$ then $a^2 = ab = ba$ and $b^2 = ba = ab$ for all a,b in S.
Consider $a^2 = ab \Rightarrow a.a^2 = a.ab \Rightarrow a.a.a = aa(aba) \Rightarrow a = aba \Rightarrow a = b \Rightarrow \rho$ is anti-symmetric. Let $a\rho b$ and $b\rho c$ then $a^2 = ab = ba$ and $b^2 = bc = cb$
Consider $a^2 = ab = (bab)b = bab^2 = babc = ac.$
Similarly $a^2 = ca = ac \Rightarrow a\rho c$ .Hence ρ is transitive.
Let $a \rho b \Rightarrow a^2 = ab = ba \Rightarrow a^2c^2 = abc^2 = bac^2 \Rightarrow a^2c^2 = abcc = bacc.$
$\Rightarrow a^2c^2 = (ac)(bc) = (bc)(ac) \Rightarrow ac \rho bc$ similarly $ca \rho cb.$
Let $a \rho b \Rightarrow a^2 = ab = ba$,Consider $(a+c)^2 = (a+c)(a+c)$
$(a+c)^2 = a^2+ac+ca+c^2 = ab+ac+ca+c^2 = a(b+c) + c(bab) + c.c = a(b+c) + c.b.a^2+c.c$
$= a(b+c) + cb.aa+cc = a(b+c) + cba(bab)+cc = a(b+c) + c.b.b.b+c.c$
$= a(b+c) + c.b+c.c = a(b+c) + c(b+c) = (a+c)(b+c)$

Similarly we can prove that $(c+a)^2 = (c+a)(c+b)$
$\Rightarrow (a+c) \rho (b+c)$ and $(c+a) \rho (c+b)$.Therefore (S,+,., ρ) is a partially ordered semiring.

Theorem 3.12. Let (S,+,.) be an anti-inverse idempotent semiring then S is distributive.
Proof: Given that (S,+,.) be an anti-inverse idempotent semiring. Thus $aba = b$ for all a, b in S.

To prove that S is distributive.It is enough to show that '+' distributive over '.'.
$(a.b) + c = (a+c).(b+c)$ for all a,b,c in S.

Consider $(a+c).(b+c) = (a+c).b+(a+c).c = (a.b)+(c.b)+(a.c)+(c.c) = (a.b) +(c.b)+(a.c)+c = (a.b)+(c.b)+(cac.aca)+c = (a.b) +(c.b) +(c.c.c.a)+c = (a.b) + (c.b) + (c.a) +c = (a.b) +bcb.cbc + (aca)(cac) + c = (a.b) + bc(bb)cbc + (aca)(aca)c + c = (a.b) + (bcb)(bcb)c + (aca)ac(aa)c + c = (a.b) + c.c.c + cac(aa)c + c= (a.b) + c + c.aca.c + c = (a.b) + c +c.c.c +c = (a.b) + c +c +c = (a.b) +c$

Similarly we can prove the other distribution condition
$c+(a.b) =(c+a).(c+b)$ forall a,b,c in S .Hence S is distributive.

4. Conclusions

If (S,+,.) is an anti-inverse semiring then (S,+,.) separative, (S,.) is normal, (S,+.) is a Boolean semiring, (S,+,., ρ) is a partially order semiring and also S satisfies some structural and multiplicative properties of semirnigs.

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