

A Study on Domination Sets in Vague Graphs with Novel Applications

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Received 15 August 2017; accepted 15 December 2017

Abstract. Theoretical concepts of graphs are highly utilized by computer science applications. Especially in research areas of computer science such as data mining, image segmentation, clustering, image capturing and networking. Fuzzy graph theory has numerous applications in modern sciences and technology such as neural networks, artificial intelligence and decision making. A vague graph is a generalized structure of a fuzzy graph that gives more precision, flexibility, and compatibility to a system when compared with systems that are designed using fuzzy graphs. In this paper, we study some results on point set dominating set and its number of vague graphs and semi complete vague graphs. Likewise, we introduce the bounds with other domination parameters of vague graphs. Finally, we give some applications of point set dominating sets in the other sciences.

Keywords: Vague graph, point set domination set, semi complete vague graph

AMS Mathematics Subject Classification (2010): 05C99, 05C22

1. Introduction

Graph theory is an extremely useful tool in solving combinatorial problems in different areas including geometry, algebra, number theory, topology, operations research, biology and social systems. Fuzzy graph theory is finding an increasing number of application in modeling real time systems where the level of information inherent in the system varies with different levels of precision. In 1965, Zadeh [19] first proposed the theory of fuzzy sets. Gau and Buehrer [3] proposed the concept of vague set in 1993, by replacing the value of an element in a set with a subinterval of $[0,1]$. Namely, a true-membership function $t_v(x)$ and a false membership function $f_v(x)$ are used to describe the boundaries of the membership degree. The first definition of fuzzy graphs was proposed by Kafmann [4] in 1993, from Zadeh's fuzzy relations [19]. Ramakrishna [7] introduced the concept of vague graphs and studied some of their properties. Akram et al. [1] defined vague hypergraphs. Somasundaram and Somasundaram [17] investigated domination in fuzzy graphs.

Domination in intuitionistic fuzzy graphs introduced by Parvathi and Thamizhendhi [6]. Gani et al. [5] studied new concepts of domination in intuitionistic fuzzy graphs. Borzooei et al. [2] defined domination in vague graphs. Rashmanlou et al. [8,9,10,11,12,13,14,15, 16] investigated some basic results on bipolar fuzzy graphs and vague graphs. In this paper, we study some results on point set dominating set and its number of vague graphs and semi complete vague graph. Also, we introduce the bounds with other domination parameters of vague graphs. Lots of works have been done on fuzzy and related graphs by several researchers [20-43].

2. Preliminaries

In this section, we review briefly some definitions in vague graphs and introduce some new notations. A graph is an ordered pair $G = (V, E)$, where V is the set of vertices of G and E is the set of edges of G . A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ with $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$, where V is a finite non-empty set and \wedge denote minimum.

A vague set A in an ordinary finite non-empty set X , is a pair (t_A, f_A) , where $t_A : X \rightarrow [0,1]$, $f_A : X \rightarrow [0,1]$ are true and false membership functions, respectively such that $0 \leq t_A(x) + f_A(x) \leq 1$, for all $x \in X$. Note that $t_A(x)$ is considered as the lower bound for degree of membership of x in A and $f_A(x)$ is the lower bound for negative of membership of x in A . So, the degree of membership of x in the vague set A , is characterized by the interval $[t_A(x), 1 - f_A(x)]$. Hence, a vague set is a special case of interval-valued sets studied by many mathematicians and applied in many branches of mathematics.

It is worth to mention here that interval valued fuzzy sets are not vague sets. In interval-valued fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the evidence for x only, without considering evidence against x . In vague sets both are independently proposed by the decision maker. This makes a major difference in the judgment about the grade of membership. A vague relation is a generalization of a fuzzy relation. Let X and Y be ordinary finite non-empty sets. We call a vague relation to be a vague subset of $X \times Y$, that is an expression R defined by

$$R = \{ \langle (x, y), t_R(x, y), f_R(x, y) \rangle \mid x \in X, y \in Y \}$$

where $t_R : X \times Y \rightarrow [0,1]$, $f_R : X \times Y \rightarrow [0,1]$, which satisfies the condition $0 \leq t_R(x, y) + f_R(x, y) \leq 1$, for all $(x, y) \in X \times Y$. (See [7])

Definition 2.1. [7] A vague graph is defined to be a pair $G = (A, B)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that $t_B(xy) \leq \min(t_A(x), t_A(y))$ and $f_B(xy) \geq \max(f_A(x), f_A(y))$, for each edge $xy \in E$. A vague graph G is said to be connected if there is a path between any two vertices of G . The underlying crisp graph of a vague graph $G = (A, B)$, is the graph $G^* = (V, E)$

A Study on Domination sets in Vague Graphs with Novel Applications

where $V = \{v : t_A(v) > 0 \text{ and } f_A(v) > 0\}$ and $E = \{uv : t_B(uv) > 0, f_B(uv) > 0\}$. V is called the vertex set and E is called the edge set. A vague graph may be also denoted as $G = (V, E)$.

A vague graph G is said to be strong if $t_B(v_i v_j) = \min\{t_A(v_i), t_A(v_j)\}$ and $f_B(v_i v_j) = \max\{f_A(v_i), f_A(v_j)\}$, for every edge $v_i v_j \in E$. A vague graph G is said to be complete if $t_B(v_i v_j) = \min\{t_A(v_i), t_A(v_j)\}$ and $f_B(v_i v_j) = \max\{f_A(v_i), f_A(v_j)\}$, for all $v_i, v_j \in V$.

A vague graph G is said to be n -partite (multipartite) if the vertex set V can be partitioned into n subsets V_1, V_2, \dots, V_n such that: (i) For every $v_i, v_j \in V_i$ and $1 \leq i, j \leq n$, we have

$$t_B(v_i v_j) = 0 \text{ and } f_B(v_i v_j) = 0.$$

(ii) For every $v_i \in V_i, v_j \in V_j$ and $1 \leq i, j \leq n$, we have

$$t_B(v_i v_j) = 0, f_B(v_i v_j) > 0 \text{ or } t_B(v_i v_j) > 0, f_B(v_i v_j) = 0.$$

Definition 2.2. [2] Let $G = (A, B)$ be a vague graph.

(i) The cardinality of G is defined to be

$$|G| = \left| \sum_{v_i \in V} \frac{1+t_A(v_i)+f_A(v_i)}{2} + \sum_{v_i v_j \in E} \frac{1+t_B(v_i v_j)-f_B(v_i v_j)}{2} \right|.$$

(ii) The vertex cardinality of G is defined by

$$|V| = \sum_{v_i \in V} \frac{1+t_A(v_i)-f_A(v_i)}{2}, \text{ for all } v_i \in V.$$

(iii) The edge cardinality of G is defined by

$$|E| = \sum_{v_i v_j \in E} \left(\frac{1+t_B(v_i v_j)-f_B(v_i v_j)}{2} \right), \text{ for all } v_i v_j \in E.$$

(iv) The number of vertices (the cardinality of V) is called the order of a vague graph and is denoted by

$$O(G) = \sum_{v_i \in V} \left(\frac{1+t_A(v_i)-f_A(v_i)}{2} \right), \text{ for all } v_i \in V.$$

(v) The number of edges (the cardinality of E) is called the size of a vague graph and is denoted by

$$S(G) = \sum_{v_i v_j \in E} \left(\frac{1+t_B(v_i v_j)-f_B(v_i v_j)}{2} \right), \text{ for all } v_i v_j \in E.$$

The complement of $G = (A, B)$ is a vague graph $\bar{G} = (\bar{A}, \bar{B})$ where $\bar{A} = A$,

$$\overline{t_B}(v_i v_j) = \min(t_A(v_i), t_A(v_j)) - t_B(v_i v_j) \text{ and}$$

$$\overline{f_B}(v_i v_j) = \max(f_A(v_i), f_A(v_j)) - f_B(v_i v_j).$$

Definition 2.3. [2] An edge $e = uv$ of a vague graph is called an effective edge if $t_B(uv) = \min\{t_A(u), t_A(v)\}$ and $f_B(uv) = \max\{f_A(u), f_A(v)\}$.

Definition 2.4. [2] Let $G = (V, E)$ be a vague graph.

(i) The neighbourhood of any vertex v is defined as $N(v) = (N_t(v), N_f(v))$, where $N_t(v) = \{w \in V; t_B(vw) = t_A(v) \wedge t_A(w)\}$ and $N_f(v) = \{w \in V; f_B(vw) = f_A(v) \vee f_A(w)\}$. $N[v] = N(v) \cup \{v\}$ is called the closed neighbourhood of v .

(ii) The neighbourhood degree of a vertex is defined as $d_N(v) = (d_{N_t}(v), d_{N_f}(v))$,

where $d_{N_t}(v) = \sum_{w \in N(v)} t_A(w)$ and $d_{N_f}(v) = \sum_{w \in N(v)} f_A(w)$. The minimum

neighbourhood degree is defined as $\delta_N(G) = (\delta_{N_t}(v), \delta_{N_f}(v))$, where

$$\delta_{N_t}(v) = \wedge \{d_{N_t}(v) : v \in V\} \text{ and } \delta_{N_f}(v) = \wedge \{d_{N_f}(v) : v \in V\}.$$

(iii) The effective degree of a vertex v is defined to be sum of the effective edges incident at v , and denoted by $d_E(v)$. The minimum effective degree of G is

$$\delta_E(G) = \wedge \{d_E(v) | v \in V\}.$$

Definition 2.5. [2] Let $G = (V, E)$ be a vague graph and $u, v \in V$.

(i) We say that u dominated v in G if there exist a strong arc between them. A subset $D \subseteq V$ is said to be dominating set in G if for every $v \in V - D$, there exist u in D such that u dominated v . The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by $\gamma(G)$. The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol $\Gamma(G)$.

(ii) A subset $D \subseteq V$ is said to be total dominating set in G if every vertex in V is dominated by a node in D . The minimum cardinality of all total dominating set is called total domination number and denoted by $\gamma_t(G)$.

(iii) An independent set of a vague graph G is a subset S of V such that no two vertices of S are adjacent in G .

Definition 2.6. [2] A bipartite vague graph G is said to be complete bipartite vague graph if $t_B(v_i v_j) = t_A(v_i) \wedge t_A(v_j)$ and $f_B(v_i v_j) = f_A(v_i) \vee f_A(v_j)$, for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by K_{v_1, v_2} .

Definition 2.7. [2] Let $G = (V, E)$ be a connected vague graph with effective edges. Then G is said to be semi complete vague graph if every pair of vertices have common neighbor in G . A vague graph G is said to be purely semi-complete vague graph if and only if G is semi complete vague graph but not complete vague graph.

3. Point set domination in the vague graphs

Domination in graphs has many applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that every one is serviced. Concepts from domination set also appear in problems involving finding sets of representatives in monitoring communication or electrical networks, and in land surveyor must stand in order to take height measurements for an entire region. In this section, we introduce the concept of point set dominating set in vague graphs and semi complete vague graphs and obtain some interesting results for these new parameters.

Definition 3.1. Let $G = (V, E)$ be a vague graph. A set $D \subseteq V$ is said to be a point set dominating set or a *psd*-set of G if for every $S \subseteq V - D$, there exists a node $d \in D$ such that the vague subgraph $\langle S \cup \{d\} \rangle$ induced by $S \cup \{d\}$ is a connected strong vague graph. The point set domination number $\gamma_{ip}(G)$ of G is the minimum scalar cardinality of point set dominating sets.

Example 3.2. Consider vague graph G in Figure 1.

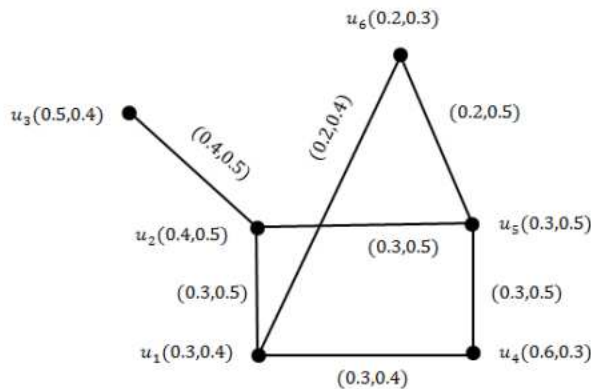


Figure 1: Vague graph G

By routine calculation we have: $|u_1| = 0.45$, $|u_2| = 0.45$, $|u_3| = 0.55$, $|u_4| = 0.65$, $|u_5| = 0.4$, $|u_6| = 0.45$. γ_{ip} -set is $\{u_3, u_4, u_6\}$ and $\gamma_{ip}(G) = 1.65$.

Theorem 3.3. For any vague graph G , $\min\{|v_i|\} \leq \gamma_{ip}(G) < p$.

Proof: If G is a complete vague graph, then $\gamma_{ip}(G)$ -set contains only one vertex and we know that $\gamma_{ip}(G)$ -set have at least one vertex. That is $\min\{|v_i|\} \leq \gamma_{ip}(G)$. Also, suppose that S be the psd -set and $V - S$ is not an empty set. Then S contain at most $n-1$ vertices. That is the cardinality of S is less than P . Therefore, $\gamma_{ip}(G) < P$. Hence, we get $\min\{|v_i|\} \leq \gamma_{ip}(G) < P$.

Remark 3.4. (i) $\gamma_{ip}(K_n) = \min\{|v_i|\}$, where $v_i \in V$.

(ii) $\gamma_{ip}(K_{v_1, v_2}) = \min\{|v_{1i}| + |v_{2j}|\}$, where $v_i \in V_1$ and $v_j \in V_2$.

Theorem 3.5. Every psd -set is a dominating set but the converse is not correct in general.

Proof: Let S be psd -set. By the definition, it is always a dominating set of G but may not minimum. For the minimal dominating set D , there exist a vertex $d \in D$ such that $\langle S \cup \{d\} \rangle$ may not be connected and $S \subseteq V - D$. That is, converse need not be true.

Example 3.6. In Figure 1, γ -set $D = \{u_1, u_2\}$, $\gamma(G) = 0.9$, and γ_{ip} -set is $\{u_3, u_4, u_6\}$. Here, γ -set is not $\gamma_{ip}(G)$ -set. Since, for the set $S = \{u_3, u_6\} \subseteq V - D$, both $\langle S \cup \{u_1\} \rangle$ and $\langle S \cup \{u_2\} \rangle$ are disconnected. Also, $\gamma(G) < \gamma_{ip}(G)$.

Theorem 3.7. Let $G = (V, E)$ be a vague graph and D is psd -set of G . Then $\langle V - D \rangle$ is a proper sub graph of a component H of G .

Proof: Suppose there exist vertices u and v belonging to two different components of G . Since D is a psd -set of G , there exist $w \in D$ such that $\langle \{u, v, w\} \rangle$ is strongly connected vague graph which is contradiction to our assumption, i.e. $V - D \subseteq V(H)$, for some component H of G . Also, $D \cap V(H) \neq \emptyset$ which implied $\langle V - D \rangle$ is a proper sub graph of H .

Theorem 3.8. Let $G = (V, E)$ be a vague graph. If S is a psd -set and $u, v \in V - S$, then there are at most two strong arcs between u and v .

Proof: Let $D = \{u, v\}$, u and v are not adjacent, and S is the psd -set. Then, there exist a vertex x in S such that the sub graph $\langle \{u, v, x\} \rangle$ is strongly connected. This implies there are at most two strong arcs between the vertices u and v .

Theorem 3.9. Let $G = (V, E)$ be a strongly connected vague graph and S is the dominating set of G which is not psd -set in G . Then the minimal γ -set contains at least two vertices.

A Study on Domination sets in Vague Graphs with Novel Applications

Proof: Since S is not a psd -set, there is a $D \subseteq V - S$ such that $\langle D \cup \{v\} \rangle$ is not connected for any $v \in S$. Hence, $\langle D \rangle$ is not connected and therefore it has at least two components. So, at least two vertices in S are not dominated by a single vertex from S , which implies γ -set contains at least two vertices.

Theorem 3.10. Let $G = (V, E)$ be the purely semi complete vague graph and has a unique path between any pair of vertices in G . Then the intersection of all psd -sets in G is a singleton set.

Proof: Since G is purely semi complete vague graph, G is a union of edge disjoint triangles having a common vertex, say u . It follows that any psd -set contains the vertex u . Also, $\{u\}$ itself is psd -set. Hence, their intersection is $\{u\}$.

Remark 3.11. The converse of the above theorem is not true i.e. if the intersection of all psd -sets in G is a singleton set, then G need not be a purely semi complete vague graph with unique path between any pair of vertices in G . Following example shows this subject.

Example 3.12. Consider purely semi-complete vague graph G in Figure 2.

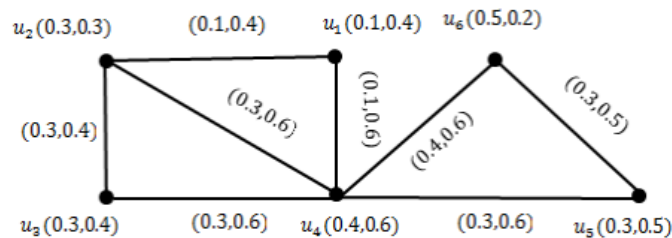


Figure 2: Purely semi-complete vague graph G

Here, all the psd -sets include u_4 . But there are two different shortest paths $\{u_4, u_1, u_2\}$ and $\{u_4, u_3, u_2\}$ between the vertices u_2 and u_4 .

Theorem 3.13. Let $G = (V, E)$ be the purely semi complete vague graph with n vertices and S be the dominating set with number of vertices is greater than or equal to $n - 2$. Then, S is psd -set in G .

Proof: Let $D \subseteq V - S$ and assume that S has $n - 2$ vertices. Then either D is singleton or contain two vertices. Let $D = \{v\}$. Since S is dominating set, there is a $u \in S$ such that v is adjacent to u . That is $\langle D \cup \{u\} \rangle$ is connected. Now, let $D = \{v_1, v_2\}$. If v_1 and v_2 are adjacent, then there is a $u \in S$ such that $\langle D \cup \{u\} \rangle$ is connected. Suppose that v_1 and v_2 are not adjacent. Then, by the nature of S , there is a $u \in S$ such that $\langle D \cup \{u\} \rangle$ is connected. Thus S is psd -set in G .

Theorem 3.14. *Let $G = (V, E)$ be the purely semi complete vague graph and S is a dominating set in G . Then, S is psd -set if and only if for any independent vertex set $B \subseteq V - S$, there is a $v \in S$ such that every vertex of B is strongly adjacent to v in G .*

Proof: Let S be a psd -set in G , B be any independent vertex set, and $B \subseteq V - S$. Then, there is a $v \in S$ such that $\langle S \cup \{v\} \rangle$ is strongly connected. Since no two elements in B are adjacent, by previous theorem there are two strong arcs between two vertices. Therefore, G is purely semi complete vague graph and S is a dominating set in G . Conversely, suppose S is a dominating set in G and for any independent vertex set $B \subseteq V - S$, there is a $v \in S$ such that every vertex of B is strongly adjacent to v in G . Let $D \subseteq V - S$ and $\langle D \rangle$ is connected. Then, S is psd -set. Otherwise, let there are more than one components of D . Then $B = \{v_1, v_2, \dots, v_n\}$ is an independent set in G . Now by hypothesis, since each v_i is adjacent to v in G , then there exist $v \in S$ such that $\langle B \cup \{v\} \rangle$ is connected. Therefore, $\langle D \cup \{v\} \rangle$ is connected. Hence, S is psd -set in G .

Definition 3.15. *Let $G = (V, E)$ be a vague graph. A set $D \subseteq V$ is said to be a set-domination set if for every set $T \subseteq V - D$, there exists a non-empty set $S \subseteq D$ such that the vague induced sub graph $\langle S \cup T \rangle$ is connected. The minimum cardinality taken over all set-domination sets is called set-domination number and denoted by $\gamma_s(G)$.*

Example 3.16. *In Example 3.2, it is easy to see that $\{u_3, u_4, u_5\}$ is a set-domination set.*

Theorem 3.17. *Let $G = (V, E)$ be a vague graph with effective edges. Then, $\gamma(G) \leq \gamma_s(G) \leq \gamma_{ip}(G)$.*

Proof: Let D be a point set dominating set of a vague graph. For every set $T \subseteq V - D$ there exist a non-empty singleton set $\{v\} \in S \subseteq D$ such that the sub graph $\langle S \cup T \rangle$ is connected. Thus, D is a set-domination set. So, $\gamma_s(G) \leq \gamma_{ip}(G)$. Now, let D be a set-domination set of a vague graph G . Then, for any set $\{x\} = T \subseteq V - D$, there exist a non-empty singleton set $\{v\} \in S \subseteq D$ such that the sub graph $\langle S \cup T \rangle$ is connected. Thus, D is a domination set. So, $\gamma(G) \leq \gamma_s(G)$. Therefore, we have $\gamma(G) \leq \gamma_s(G) \leq \gamma_{ip}(G)$.

4. Application of point set dominating sets in vague graphs

In this section, we give some applications of point set dominating sets in the other sciences.

4.1. Modeling social networks

Point set dominating sets can be used in modeling social networks and studying the

A Study on Domination sets in Vague Graphs with Novel Applications

dynamics of relations among numerous individuals in different domains. A social network is a social structure made of individuals (or groups of individuals), which are connected by one or more specific types of interdependency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the works of Kelleher and Cozzens, social networks are modeled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs.

4.2. Social groups

It is observed that in social group some people can influence others and it can happen only when there is a strong relationship between them. Now, let us consider a vague graph of a social group as given in Figure 3.

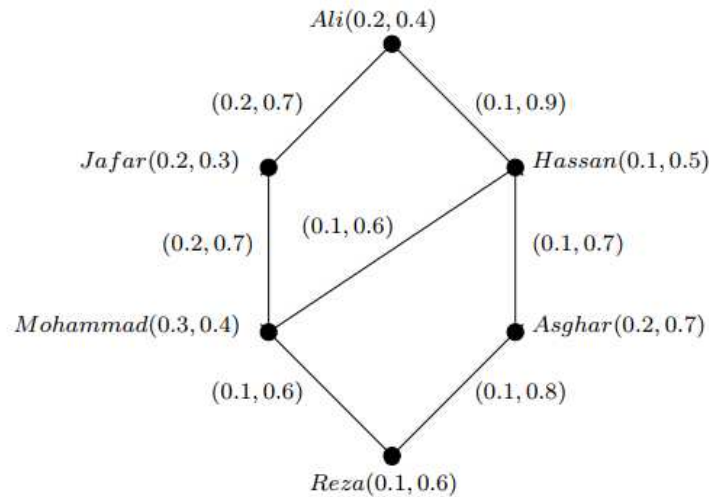


Figure 3: Vague graph G of social relation

The nodes are depicting the degree of power of a person belongs to a set of social group. The degree of power of a person is defined in terms of its true membership and false membership. Degree of true membership can be interpreted as how much power a person possess and false membership can be interpreted as how much power a person losses. *Ali* has 20% power within the social group but he losses 40% power in the same group. In this example, $\{Ali, Jafar\}$, $\{Asghar, Hassan\}$, $\{Jafar, Mohammad\}$, and $\{Mohammad, Reza\}$ are strong edges. So, we can say there is a strong relation between *Ali* and *Jafar*. It is true for *Asghar, Hassan* and *Jafar, Mohammad* too. Hence, we see that *Jafar* dominates *Ali*, *Hassan* dominates *Asghar*, and *Mohammad* dominates *Jafar*. It is clear that $\{Ali, Mohammad, Asghar\}$,

$\{Ali, Mohammad, Asghar, Reza\}$, and $\{Ali, Mohammad, Reza, Hassan\}$ are dominating sets. By routine calculation we have $|Ali| = 0.4$, $|Hassan| = 0.3$, $|Reza| = 0.25$, $|Jafar| = 0.45$, and $|Asghar| = 0.25$.

γ_{ip} -set is $\{Jafar, Ali, Hassan\}$. The edges of a vague graph show the influence of one person to another person. The degree of true membership and false membership of edges can be interpreted as the percentage of true membership and false membership influence. For example, *Hassan* follows 10% Mohammad's suggestions but he/she does not follows 60% his/her suggestions. Therefore, domination can help us to find people who have strong relation in a social group.

4.3. Social network theory

The online social network has been developed significantly in the recent years as a medium of communication, sharing the information and spreading the influence. Most of the current research has been carried out on understanding the property of how the online social network has been utilized in disseminating the information and ideas. The problem of utilizing the online social network for solving the social problems in the physical world such as drinking, smoking, and drug problems are all explored well. The dominating set plays a vital role in analyzing the effect on a real online social network data set through simulation. The dominating set concept can be applied to the social network graph to determine the amount of positive influence that is possessed by an individual as well as its impact to their related neighbor. This dominating positive degree threshold can spread the positive educational influence throughout the entire community in the social network.

In a social setup people may have both positive as well as negative impact on each other and persons can play different roles as they are affected by their peers. The positive or negative influence in social issues can move in two directions. That is, a positive individual can convert into a negative individual and can move back and forth between these two states for multiple times. A vague undirected graph $G = (V, E, C)$ is used to denote the online social network, because friendship in an online social network are usually bidirectional as shown in below.

Online social network can be represented as a graph of relationship with individuals representing the nodes of a graph (V), the social interactions as edges (E) and C is the compartment vector that saves the compartment of each node. The compartment part of a node decides whether the social issue(s) of an individual has positive or negative impact on its neighbors.

5. Conclusion

It is well known that graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological, and social systems. In general graph theory has a wide range of applications in diverse fields. In this paper, we studied some results on point set dominating set and its number of vague graphs and semi complete vague graph. Also, we introduced the bounds with other domination parameters of vague graphs.

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A Study on Domination sets in Vague Graphs with Novel Applications

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