

On Decompositions of $(r^*g^*)^*$ Closed Set in Topological Spaces

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Abstract. The aim of this paper is to obtain decompositions of $(r^*g^*)^*$ closed set. The concept of $(r^*g^*)^*$ locally closed sets and $(r^*g^*)^*$ locally continuous functions are introduced and some of their properties are investigated. Furthermore the notions of P^* sets, P^{**} sets, Q^{**} sets, W^* sets and A^* sets are introduced and are used to obtain the decompositions of $(r^*g^*)^*$ closed sets.

Keywords: $(r^*g^*)^*$ closed set, $(r^*g^*)^*$ closure, $(r^*g^*)^*$ continuous functions, $(r^*g^*)^*$ irresolute functions, $(r^*g^*)^*$ open sets.

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1. Introduction

Levin [10] introduced the concept of generalized closed set in topological spaces. The concept of locally closed sets in a topological space was introduced by Bourbaki [4]. Ganster and Reilly [5] further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. Balachandran et al. [3] introduced the concept of generalized locally closed sets and GLC – continuous functions and investigated some of their properties. Arockiarani, Balachandran and Ganster [2] introduced regular generalized locally closed sets and RGL- continuous functions. The Authors [12] have already introduced $(r^*g^*)^*$ closed sets and investigated some of their properties. The aim of this paper is to introduce $(r^*g^*)^*$ locally closed set and $(r^*g^*)^*$ locally continuous function and investigate some of their properties. Furthermore the notions of P^* sets, P^{**} sets, Q^{**} sets, W^* sets and A^* sets are used to obtain the decompositions of $(r^*g^*)^*$ closed sets.

2. Preliminaries

Definition 2.1. A subset A of a Topological space X is called

- 1) A generalized closed set (g -closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

- 2) A regular generalized closed set (rg-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
- 3) $A(r^*g^*)^*$ closed set [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is r^*g^* - open. The complement of $(r^*g^*)^*$ closed set is $(r^*g^*)^*$ open.
- 4) A locally closed set [5] if $A = S \cap F$ where S is open and F is closed.
- 5) A generalized locally closed set [3] if $A = S \cap F$ where S is g-open and F is g-closed.
- 6) A glc^* -set [3] if $A = S \cap F$ where S is g-open and F is closed.
- 7) A glc^{**} -set [3] if $A = S \cap F$ where S is open and F is g- closed.
- 8) A regular generalized locally closed set [2] is $S = G \cap F$ where G is rg-open and F is rg-closed in (X, \mathfrak{J}) .
- 9) A $rglc^*$ [2] if there exists a rg-open set G and a closed set F of (X, \mathfrak{J}) such that $S = G \cap F$.
- 10) A $rglc^{**}$ [2] if there exists an open set G and a rg-closed set F such that $B = G \cap F$.

Definition 2.2. A subset S of a topological space is called a

1. t set [17] if $int(S) = int(cl(S))$.
2. t^* set [7] if $cl(S) = cl(int(S))$.
3. α^* set if [15] $int(S) = int(cl(int(S)))$.
4. C set [16] if $S = G \cap F$ where G is open and F is a t set.
5. Cr set [16] if $S = L \cap M$ where L is rg open and M is a t set.
6. Cr^* set [16] if $S = L \cap M$ where L is rg open and M is a α^* set.
7. A set if [18] $S = G \cap F$ where G is open and F is a regular closed set.

Definition 2.3. Let X be a Topological space. Let A be a subset of X . $(r^*g^*)^*$ closure [14] of A is defined as the intersection of all $(r^*g^*)^*$ closed sets containing A .

Definition 2.3. A function $f : (X, \mathfrak{J}) \rightarrow (Y, \sigma)$ is called

- (i) g - continuous [10] if $f^{-1}(V)$ is g closed in (X, \mathfrak{J}) for every closed set V of (Y, σ) .
- (ii) $(r^*g^*)^*$ -continuous [13] if the inverse image of every closed set in (Y, σ) is $(r^*g^*)^*$ -closed in (X, \mathfrak{J})
- (iii) $(r^*g^*)^*$ -irresolute map [13] if $f^{-1}(V)$ is a $(r^*g^*)^*$ -closed set in (X, \mathfrak{J}) for every $(r^*g^*)^*$ closed set V of (Y, σ) .
- (iv) LC -continuous [5] if $f^{-1}(V)$ is a locally closed set in (X, \mathfrak{J}) for every open set V of (Y, σ) .
- (v) $G LC$ -continuous [3] if $f^{-1}(V)$ is a gl -closed set in (X, \mathfrak{J}) for every open V of (Y, σ) .
- (vi) Rgl continuous [2] if $f^{-1}(V)$ is a rgl closed set in (X, \mathfrak{J}) for every open V of (Y, σ) .

3. $(r^*g^*)^*$ locally closed sets

Definition 3.1. A Subset S of (X, \mathfrak{J}) is called $(r^*g^*)^*$ Locally closed if $S = A \cap B$ where A is $(r^*g^*)^*$ open and B is $(r^*g^*)^*$ closed.

Example 3.2. Let $X = \{a, b, c\}$. Let $\mathfrak{J} = (\emptyset, X, \{a\}, \{b\}, \{a, b\})$.

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Closed sets are $\{\emptyset, X, \{c\}, \{b,c\}, \{a,c\}\}$

$(r^*g^*)^*$ closed sets are $\{\emptyset, X, \{c\}, \{b,c\}, \{a,c\}\}$

$(r^*g^*)^*$ open sets are $\{\emptyset, X, \{a,b\}, \{a\}, \{b\}\}$

Now $\{a\} = \{a,b\} \cap \{a,c\}$ where $\{a,b\}$ is $(r^*g^*)^*$ open and $\{a,c\}$ $(r^*g^*)^*$ closed and hence $\{a\}$ is a $(r^*g^*)^*$ locally closed set.

Here $\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}$ are $(r^*g^*)^*$ Locally closed sets.

Definition 3.3. A Subset S of (X, \mathfrak{T}) is called $(r^*g^*)^*$ Locally $*$ closed if $S = A \cap B$ where A is $(r^*g^*)^*$ open and B is closed.

Example 3.4. In Example 3.2 $\{b\} = \{a,b\} \cap \{b,c\}$ is $(r^*g^*)^*$ Locally $*$ closed.

Definition 3.5. A subset S of (X, \mathfrak{T}) is called $(r^*g^*)^*$ locally $**$ closed if $S = A \cap B$ where A is open and B is $(r^*g^*)^*$ closed.

Example 3.6. In Example 3.2 $\{a\} = \{a,b\} \cap \{a,c\}$ is $(r^*g^*)^*$ locally $**$ closed.

Remark 3.7. Every closed set is $(r^*g^*)^*$ locally closed set.

Theorem 3.8.

- (i) Every Locally closed sets is $(r^*g^*)^*$ locally closed.
- (ii) Every g^* locally closed set is $(r^*g^*)^*$ locally closed
- (iii) Every $(r^*g^*)^*$ locally closed set is gpr locally closed
- (iv) Every $(r^*g^*)^*$ locally closed set is rwg locally closed
- (v) Every $(r^*g^*)^*$ locally closed set is rg locally closed

Proof:

(i) Let $S = A \cap B$ where A is open and B is closed in X . But every open set is $(r^*g^*)^*$ open and every closed set is $(r^*g^*)^*$ closed and hence S is $(r^*g^*)^*$ locally closed set.

(ii) Proof follows from the fact that every g^* closed set is $(r^*g^*)^*$ closed set [12].

(iii) Proof follows from the fact that every $(r^*g^*)^*$ closed set is gpr closed set [12].

(iv) Proof follows from the fact that every $(r^*g^*)^*$ closed set is rwg closed set [12].

(v) Proof follows from the fact that every $(r^*g^*)^*$ closed set is rg closed set [12].

The converse of the above statements need not true as seen from the following example.

Example 3.9.

(i) Let $X = \{a, b, c\}$. Let $\mathfrak{T} = \{\emptyset, X, \{c\}, \{b,c\}\}$

Closed sets are $\{\emptyset, X, \{a\}, \{a,b\}\}$

$(r^*g^*)^*$ closed sets are $\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$

$(r^*g^*)^*$ open sets are $\{\emptyset, X, \{b,c\}, \{c\}, \{b\}\}$

Here $\{a,c\}$ is $(r^*g^*)^*$ locally closed set but not locally closed set.

(ii) Let $X = \{a, b, c\}$, $\mathfrak{S} = \{\emptyset, X, \{a\}\}$

Closed sets are $\{\emptyset, X, \{a\}\}$

Here $\{b\}$ is not a g^* locally closed set but it is a $(r^*g^*)^*$ closed set.

(iii) let $X = \{a, b, c, d\}$, $\mathfrak{S} = (\emptyset, X, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\})$

Closed sets are $\{\emptyset, X, \{b, c, d\}, \{b, d\}, \{b, c\}, \{b\}\}$

Here $\{c, d\}$ is gpr closed set but not $(r^*g^*)^*$ closed set.

(iv) In the above example $\{a, c, d\}$ is rwg closed set but not $(r^*g^*)^*$ closed set.

(v) In the above example $\{c, d\}$ is rg closed set but not $(r^*g^*)^*$ closed set.

Remark 3.10.

$(r^*g^*)^*$ locally closed sets are independent of semilocally closed sets, α locally closed sets, wg locally closed sets and the following examples support our statement.

Example 3.11.

Let $X = \{a, b, c, d\}$, $\mathfrak{S} = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Here $\{a\}$ is $(r^*g^*)^*$ locally closed but not semi locally closed set.

Let $X = \{a, b, c, d\}$, $\mathfrak{S} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}\}$. Here $\{c, d\}$ is semi locally closed set but not $(r^*g^*)^*$ locally closed set.

Example 3.12. Let $X = \{a, b, c, d\}$, $\mathfrak{S} = \{\emptyset, X, \{a, c\}, \{a, d\}, \{a\}, \{a, c, d\}\}$

Here $\{a, c\}$ is not α locally closed set but $\{a, c\}$ is $(r^*g^*)^*$ locally closed set

Here $\{c, d\}$ is α locally closed set but $\{c, d\}$ is not $(r^*g^*)^*$ locally closed.

Example 3.13. From the above example, $\{c, d\}$ is wg locally closed set but $\{c, d\}$ is not $(r^*g^*)^*$ locally closed set.

Let $X = \{a, b, c, d\}$, $\mathfrak{S} = \{\emptyset, X, \{a, b, c\}, \{a, c, d\}, \{a, c\}, \{a\}, \{c\}\}$. Here $\{a, d\}$ is $(r^*g^*)^*$ closed set but not wg locally closed set.

Theorem 3.14. Every locally closed set is $(r^*g^*)^*$ locally* closed.

The converse need not be true as seen from the following example.

Example 3.15. In example 3.8 $\{a, c\}$ is $(r^*g^*)^*$ locally*closed but not locally closed.

Theorem 3.16. Every locally closed set is $(r^*g^*)^*$ locally** closed.

The converse need not be true as seen from the following example.

In example 3.9 $\{a, c\}$ is $(r^*g^*)^*$ locally**closed but not locally closed.

Theorem 3.17. If A is $(r^*g^*)^*$ locally closed in X and B is $(r^*g^*)^*$ open then $A \cap B$ is $(r^*g^*)^*$ locally closed in X .

Proof: Since A is $(r^*g^*)^*$ locally closed $A = P \cap Q$ where P is $(r^*g^*)^*$ Closed and Q is $(r^*g^*)^*$ open. Now $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$.

Since $(Q \cap B)$ is $(r^*g^*)^*$ open and P is $(r^*g^*)^*$ closed $A \cap B$ is $(r^*g^*)^*$ locally closed.

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Theorem 3.18. A subset S of (X, \mathfrak{T}) is $(r^*g^*)^*$ Locally closed $(X - S)$ is the union of a $(r^*g^*)^*$ open and a $(r^*g^*)^*$ closed set.

Proof: If S is $(r^*g^*)^*$ locally closed then $S = P \cap Q$ where P is $(r^*g^*)^*$ closed and Q is $(r^*g^*)^*$ open. Now $X - S = X - (P \cap Q) = (P \cap Q)^c = P^c \cup Q^c$. Now P^c is $(r^*g^*)^*$ open & Q^c is $(r^*g^*)^*$ closed. Hence the result.

Result 3.19. The complement of a $(r^*g^*)^*$ locally closed set need not be locally closed.

Example 3.20. Let $X = \{a, b, c\}$, $\mathfrak{T} = \{\emptyset, X, \{c\}, \{b, c\}\}$.

Closed sets are $\{\emptyset, X, \{a, b\}, \{a\}\}$

Here $\{b\}$ is $(r^*g^*)^*$ locally closed. But its complement $\{a, c\}$ is not locally closed.

Theorem 3.21. Let A and B are subsets of (X, \mathfrak{T}) . If A is $(r^*g^*)^*$ locally closed and B is open then $A \cap B$ is $(r^*g^*)^*$ locally closed.

Proof: Let A be $(r^*g^*)^*$ locally closed in (X, \mathfrak{T}) . Then there exists an open set P and $(r^*g^*)^*$ closed set Q such that $A = P \cap Q$. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$ which is $(r^*g^*)^*$ locally closed set.

Theorem 3.22. If A is $(r^*g^*)^*$ locally closed subset of (X, \mathfrak{T}) and B is $(r^*g^*)^*$ closed then $A \cap B$ is $(r^*g^*)^*$ locally closed.

Proof: Let A be $(r^*g^*)^*$ locally closed. Then $A = P \cap Q$ where P is $(r^*g^*)^*$ open and Q is closed. Now $A \cap B = (P \cap Q) \cap B = P \cap (B \cap Q)$. Hence $A \cap B$ is $(r^*g^*)^*$ locally closed.

Theorem 3.23. If A is $(r^*g^*)^*$ locally closed subset of (X, \mathfrak{T}) and B is $(r^*g^*)^*$ open then $A \cap B$ is $(r^*g^*)^*$ locally closed.

Proof: Let A be $(r^*g^*)^*$ locally closed. Then $A = P \cap Q$ where P is $(r^*g^*)^*$ open and Q is $(r^*g^*)^*$ closed. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$. Hence $A \cap B$ is $(r^*g^*)^*$ locally closed.

4. $(r^*g^*)^*$ locally continuous functions

Definition 4.1. A function $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ is called $(r^*g^*)^*$ locally continuous, if $f^{-1}(V)$ is $(r^*g^*)^*$ locally closed in (X, \mathfrak{T}) for every open set V in (Y, σ) .

Example 4.2. Let $X = \{a, b, c\}$ and $\mathfrak{T} = \{X, \emptyset, \{c\}, \{b, c\}\}$.

Closed sets are $\{X, \emptyset, \{a, b\}, \{a\}\}$. $(r^*g^*)^*$ Closed sets are $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$. $(r^*g^*)^*$ open sets are $\{X, \emptyset, \{b, c\}, \{c\}, \{b\}\}$.

$(r^*g^*)^*$ locally closed set are $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}, \{b\}\}$.

Let $Y = \{a, b, c\}$ $\sigma = \{Y, \emptyset, \{b\}\}$

Define $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$ defined by $f(a) = a, f(b) = c, f(c) = b$.

Now $\{b\} \in \sigma$ and $f^{-1}(\{b\}) = \{c\}$ which is $(r^*g^*)^*$ locally closed in (X, \mathfrak{T}) .

Hence f is $(r^*g^*)^*$ locally continuous function.

Definition 4.3. A function $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is said to be a $(r^*g^*)^*$ locally irresolute function if $f^{-1}(V)$ is a $(r^*g^*)^*$ locally closed set in (X, \mathfrak{S}) for every $(r^*g^*)^*$ locally closed set V of (Y, σ) .

Example 4.4. Let $X = \{a, b, c\}$ $\mathfrak{S} = \{\phi, X, \{a\}\}$. Closed sets = $\{\phi, X, \{b, c\}\}$
 $(r^*g^*)^*$ closed sets are $\{\phi, X, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$
 $(r^*g^*)^*$ open sets are $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{a, c\}\}$
 $(r^*g^*)^*$ locally closed sets are $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$
 $Y = \{a, b, c\}, \sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Closed set of $Y = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$
 $(r^*g^*)^*$ closed set of Y are $\{\phi, Y, \{c\}, \{b, c\}, \{a, c\}\}$
 $(r^*g^*)^*$ open sets of Y are $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$
 $(r^*g^*)^*$ locally closed sets are $\{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$
 Here Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be defined by $f(c)=c, f(b)=a, f(a)=b$
 $f^{-1}(\{a\})=\{b\}, f^{-1}(\{b\})=\{a\}, f^{-1}(\{c\})=\{c\}$ $f^{-1}(\{a, b\})=\{a, b\}, f^{-1}(\{b, c\})=\{a, c\}, f^{-1}(\{a, c\})=\{b, c\}$
 which are $(r^*g^*)^*$ locally closed in (X, \mathfrak{S}) . Hence f is a $(r^*g^*)^*$ locally closed irresolute map.

Theorem 4.5. Every locally continuous function is $(r^*g^*)^*$ locally continuous.

Proof: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a locally continuous map. Let F be an open set in (Y, σ) . Then $f^{-1}(F)$ is locally closed in (X, \mathfrak{S}) . Since every locally closed set is $(r^*g^*)^*$ locally closed, $f^{-1}(F)$ is $(r^*g^*)^*$ locally-closed set. Therefore f is $(r^*g^*)^*$ locally continuous. The converse need not be true as seen from the following example.

Example 4.6. Let $X = \{a, b, c\}$ $\mathfrak{S} = \{\phi, X, \{c\}, \{b, c\}\}$.
 Closed set of $X = \{\phi, X, \{a, b\}, \{a\}\}$
 Locally closed sets are $\{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$
 $(r^*g^*)^*$ closed sets are $\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$
 $(r^*g^*)^*$ open sets are $\{\phi, X, \{b, c\}, \{c\}, \{b\}\}$
 $(r^*g^*)^*$ locally closed sets are $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$
 Let $Y = \{a, b, c\}, \sigma = \{\phi, Y, \{b\}\}$. Closed set of $Y = \{\phi, Y, \{a, c\}\}$
 Locally closed sets are $\{\phi, Y, \{b\}, \{a, c\}\}$. Define f by $f(c)=a, f(b)=b, f(a)=c$.
 Now $\{b\}$ is open in (Y, σ) . $f^{-1}\{b\}=\{b\}$ which is $(r^*g^*)^*$ locally closed set. Therefore f is $(r^*g^*)^*$ locally continuous. But $f^{-1}\{b\}=\{b\}$ is not locally closed in (X, \mathfrak{S}) . Hence f is not locally continuous.
 Similarly we can prove the following results.

Theorem 4.7.

- (i) Every g^* locally continuous function is $(r^*g^*)^*$ locally continuous.
- (ii) Every $(r^*g^*)^*$ locally continuous function set is gpr locally continuous function
- (iii) Every $(r^*g^*)^*$ locally continuous function set is rwg locally continuous function
- (iv) Every $(r^*g^*)^*$ locally continuous function set is rg locally continuous function.

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Definition 4.8. A function $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is called $(r^*g^*)^*$ locally $*$ continuous, if $f^{-1}(V)$ is $(r^*g^*)^*$ locally $*$ closed in (X, \mathfrak{S}) for every $V \in \sigma$.

Example 4.9. In example 4.6 the function f is $(r^*g^*)^*$ locally $*$ continuous function.

Definition 4.10. A map $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is said to be a $(r^*g^*)^*$ locally $*$ irresolute map if $f^{-1}(V)$ is a $(r^*g^*)^*$ locally $*$ closed set in (X, \mathfrak{S}) for every $(r^*g^*)^*$ locally $*$ closed set V of (Y, σ) .

Example 4.11. In example 4.4 f is $(r^*g^*)^*$ locally $*$ irresolute.

Definition 4.12. A function $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is $(r^*g^*)^*$ locally $**$ continuous, if $f^{-1}(V)$ is $(r^*g^*)^*$ locally $**$ closed in (X, \mathfrak{S}) for every V open in (Y, σ) .

In example 4.6 the function f is $(r^*g^*)^*$ locally $**$ closed continuous

Definition 4.13. A function $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is said to be a $(r^*g^*)^*$ locally $**$ irresolute function if $f^{-1}(V)$ is a $(r^*g^*)^*$ locally $**$ closed set in (X, \mathfrak{S}) for every $(r^*g^*)^*$ locally $**$ closed set V of (Y, σ) .

Example 4.14. The function f defined in example 4.4 is a $(r^*g^*)^*$ locally $**$ irresolute.

Theorem 4.15. Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a function. If f is locally continuous, then it is $(r^*g^*)^*$ locally $*$ continuous and $(r^*g^*)^*$ locally $**$ continuous.

Proof: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a locally continuous function. Let $F \in \sigma$. Then $f^{-1}(F)$ is locally closed in (X, \mathfrak{S}) . Since every locally closed set is $(r^*g^*)^*$ locally $*$ closed, $f^{-1}(F)$ is $(r^*g^*)^*$ locally $*$ closed set. Therefore f is $(r^*g^*)^*$ locally $*$ continuous. Also since every locally closed set is $(r^*g^*)^*$ locally $**$ closed, $f^{-1}(F)$ is $(r^*g^*)^*$ locally $**$ closed set. Hence f is $(r^*g^*)^*$ locally $**$ continuous.

The converse need not be true as seen from the following example.

Example 4.16. Let $X = \{a, b, c\}$ and $\mathfrak{S} = \{X, \emptyset, \{c\}, \{b, c\}\}$.

Closed sets are $\{X, \emptyset, \{a, b\}, \{a\}\}$

Locally closed sets of X are $\{X, \emptyset, \{b\}, \{a, c\}\}$.

$(r^*g^*)^*$ Closed sets of X are $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$

$(r^*g^*)^*$ open sets of X are $\{X, \emptyset, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$ locally closed set of X are $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$ locally $*$ closed set of X are $\{X, \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$ locally $**$ closed set of X are $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}\}$

Let $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ Closed set = $\{\emptyset, Y, \{b, c\}\}$

Define a mapping $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ by $f(a)=a$, $f(b)=c$, $f(c)=b$.

Here $f^{-1}\{a\}=\{a\}$ is $(r^*g^*)^*$ locally $*$ closed and $(r^*g^*)^*$ locally $**$ closed but not a locally closed set. Hence f is $(r^*g^*)^*$ locally $*$ closed continuous and $(r^*g^*)^*$ locally $**$ continuous but not locally continuous.

Theorem 4.17. Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a function. If f is $(r^*g^*)^*$ locally* continuous, then it is $(r^*g^*)^*$ locally continuous.

Proof: Let f be $(r^*g^*)^*$ locally*continuous. Let $V \in \sigma$. Then $f^{-1}(V)$ is $(r^*g^*)^*$ locally*closed $\therefore f^{-1}(V) = F \cap G$ where F is $(r^*g^*)^*$ open and G is closed. But every closed set is $(r^*g^*)^*$ closed. $\therefore G$ is $(r^*g^*)^*$ closed and.

$\therefore f^{-1}(V)$ is $(r^*g^*)^*$ locally closed. Hence f is $(r^*g^*)^*$ locally continuous.

The converse need not be true as seen from the following example.

Example 4.18. In example 4.16 Let $Y = \{a, b, c\}, \sigma = \{\emptyset, Y, \{b, c\}\}$.

Closed set $= \{\emptyset, Y, \{a\}\}$ Define f by $f(a)=b, f(c)=c, f(b)=a$.

$f^{-1}\{b, c\} = \{a, c\}$ is $(r^*g^*)^*$ locally closed and hence f is $(r^*g^*)^*$ locally continuous but $f^{-1}\{b, c\} = \{a, c\}$ is not $(r^*g^*)^*$ locally*closed. Therefore f is not $(r^*g^*)^*$ locally* continuous.

Similarly, we can prove the following theorem.

Theorem 4.19. Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a map. If f is $(r^*g^*)^*$ locally** continuous, then it is $(r^*g^*)^*$ locally continuous.

The converse need not be true as seen from the following example.

In example 4.16 let $Y = \{a, b, c\}, \sigma = \{\emptyset, Y, \{b\}\}$. Closed set $= \{\emptyset, Y, \{a, c\}\}$ define f by $f(b)=b, f(c)=a, f(a) = c$. Now $f^{-1}\{b\} = \{b\}$ is $(r^*g^*)^*$ locally closed and hence f is $(r^*g^*)^*$ locally continuous. But $f^{-1}\{b\} = \{b\}$ is not $(r^*g^*)^*$ locally** closed in X . Therefore f is not $(r^*g^*)^*$ locally** continuous.

Theorem 4.20. Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ be a map. If f is $(r^*g^*)^*$ locally* irresolute, then it is $(r^*g^*)^*$ locally continuous.

Proof: Let $V \in \sigma$. Then $V = V \cap Y$. Hence V is $(r^*g^*)^*$ locally closed in Y . Since f is $(r^*g^*)^*$ locally*irresolute, $f^{-1}(V)$ is $(r^*g^*)^*$ locally*closed. Now $f^{-1}(V) = F \cap G$, where F is $(r^*g^*)^*$ open and G is closed. But every closed set is $(r^*g^*)^*$ closed $\therefore f^{-1}(V)$ is $(r^*g^*)^*$ locally closed. Hence f is $(r^*g^*)^*$ locally continuous.

The converse need not be true as seen from the following example.

In example 4.16, let $Y = \{a, b, c\}, \sigma = \{\emptyset, Y, \{a, b\}\}$. Closed set $= \{\emptyset, Y, \{c\}\}$. Let f be defined by $f(a)=a, f(c)=b, f(b) = c$. Now $f^{-1}\{a, b\} = \{a, c\}$ is $(r^*g^*)^*$ locally closed but not $(r^*g^*)^*$ locally*closed. Hence f is $(r^*g^*)^*$ locally continuous but not $(r^*g^*)^*$ locally* irresolute.

Remark 4.21. Composition of two $(r^*g^*)^*$ locally continuous function need not be $(r^*g^*)^*$ locally continuous.

Let $X=Y=\{a, b, c, d\}$.

Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ where $\mathfrak{S} = \{\emptyset, X, \{a, c\}, \{a, d\}, \{a\}, \{a, c, d\}\}$.

$\sigma = \{\emptyset, X, \{a, b\}, \{c, d\}\}$.

Let f be defined by $f(a) = a, f(d) = c, f(c) = b, f(b) = d$

$f^{-1}\{a, b\} = \{a, c\}, f^{-1}\{c, d\} = \{d, b\}$

$f^{-1}\{a, b\}$ is $(r^*g^*)^*$ locally closed. $f^{-1}\{c, d\}$ is $(r^*g^*)^*$ locally closed. Hence

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$f : X \rightarrow Y$ is $(r^*g^*)^*$ locally continuous

let $g : (Y, \sigma) \rightarrow (Z, \eta)$ where $\eta = \{ \emptyset, X, \{a, b\} \}$ be defined by $g(c) = b, g(a) = c, g(d) = d, g(b) = a$.

$g^{-1}(\{a, b\}) = \{b, c\}$ is $(r^*g^*)^*$ locally closed and hence g is $(r^*g^*)^*$ locally continuous but $(g \circ f)^{-1}(\{a, b\}) = f^{-1}(g^{-1}(\{a, b\})) = f^{-1}(\{b, c\}) = \{c, d\}$ is not $(r^*g^*)^*$ locally closed. Hence $g \circ f$ is not $(r^*g^*)^*$ locally continuous.

The following theorem gives the condition under which the composition of two functions is $(r^*g^*)^*$ locally continuous.

Theorem 4.22. Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two function. Then

- 1) $g \circ f$ is $(r^*g^*)^*$ locally continuous if g is $(r^*g^*)^*$ locally continuous and f is $(r^*g^*)^*$ locally irresolute
- 2) $g \circ f$ is $(r^*g^*)^*$ locally irresolute if both f and g are $(r^*g^*)^*$ locally irresolute.
- 3) $g \circ f$ is $(r^*g^*)^*$ locally* continuous if g is $(r^*g^*)^*$ locally* continuous and f is $(r^*g^*)^*$ locally* irresolute.

5. Another decomposition of $(r^*g^*)^*$ closed sets

The following definitions are introduced to obtain decompositions of $(r^*g^*)^*$ closed set.

Definitions 5.1. A subset A of a topological space X is called a

- 1) P^* set if $A = L \cap M$ where L is $(r^*g^*)^*$ open and M is a t set.
- 2) P^{**} set if $A = L \cap M$ where L is $(r^*g^*)^*$ open and M is a t^* set.
- 3) Q^{**} set if $A = L \cap M$ where L is $(r^*g^*)^*$ open and M is a C set.
- 4) W^* set if $A = L \cap M$ where L is $(r^*g^*)^*$ open and M is an α^* set.
- 5) A^* set if $A = L \cap M$ where L is $(r^*g^*)^*$ open and M is a regular closed set.

Propositions 5.2.

1. Every C set is a P^* set
- 1) Every P^* set is Cr set.
- 2) Every W^* set is Cr^* set.
- 3) Every A set is A^* set.
- 4) Every A^* set is P^{**} set.
- 5) Every t set is P^* set.
- 6) Every C set is Q^{**} set.
- 7) Every α^* set is W^* set.
- 8) Every $(r^*g^*)^*$ open set is P^* set.
- 9) Every $(r^*g^*)^*$ open set is W^* set.

Remark 5.3. The converses need not be true as seen from the following examples.

Example 1. Let $X = \{a, b, c\}$ $\mathfrak{S} = \{\emptyset, X, \{a\}, \{b, c\}\}$.
Here $\{b\}$ is P^* but not C .

Example 2. Let $X = \{a, b, c\}$ $\mathfrak{S} = \{\emptyset, X, \{b\}, \{a, b\}\}$.
Here $\{b, c\}$ is a Cr set but not a P^* set.

Example 3. In example 2 $\{b, c\}$ is Cr^* but not W^* .

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Example 4. In example 2 $\{b,c\}$ is P^{**} but not A^* .

Example 5. In example 2 $\{a\}$ is A^* but not A .

Example 6. In example 2 $\{a,b\}$ is P^* but not t

Example 7. In example 1 $\{a,b\}$ is Q^{**} but not C .

Example 8. In example 2 $\{b\}$ is w^* but not α^*

Example 9. In example 2 $\{c\}$ is P^* but not $(r^*g^*)^*\text{open}$.

Example 10. In example 2 $\{a,c\}$ is W^* but not $(r^*g^*)^*\text{open}$.

REFERENCES

1. I.Arockiarani and K.Balachandran and M.Ganster, Regular generalized locally closed sets and RGL continuous functions, *Indian J. Pure and Math.*, 28(5) (1997) 661-669.
2. K.Balachandran, P.Sundaram and H.Maki, Generalized locally closed set and GLC – continuous functions, *Indian J. Pure and Math.*, 27(3) (1996) 235-244.
3. N.Bourbaki, *General Topology, Part I*. Addison–Wesley, Reading Mass, 1966.
4. M.Ganster and I.L.Reilly, Locally closed sets and LC–continuous, *Internat. J. Math. Sci.*, 12 (1989) 417-24.
5. G.Navalagi, Properties of GS-closed sets and SG closed sets in topology, *Int. J. of Communication in Topology*, 1(1) (2013) 31-40.
6. T.Indira and K.Rekha, On locally $^{**}b$ -closed sets, *Proceedings of the Heber International Conference on Applications of Mathematics and Statistics* (2012).
7. T.Indira and K.Rekha, Applications of $^{*}b$ -open sets and $^{**}b$ open sets in topological spaces, *Annals of Pure and Applied Mathematics*, 1(1) (2012) 44–56.
8. T.Indira and S.Geetha, $^{*}\text{-}g\alpha$ closed sets in topological spaces, *Annals of Pure and Applied Mathematics*, 4(2) (2013) 138-144.
9. N.Levine, Generalized closed sets in topology, *Rend. Circ. Math. Palermo*, 19(2) (1970) 89-96.
10. N.Meenakumari and T.Indira, r^*g^* closed sets in topological spaces, *Annals of Pure and Applied Mathematics*, 6(2) (2014) 125-132.
11. N.Meenakumari and T.Indira, On $(r^*g^*)^*$ closed sets in topological spaces, *International J. Science and Research*, 4(12) (2015) 23-34.
12. N.Meenakumari and T.Indira, On $(r^*g^*)^*$ continuous maps in topological spaces, *International Journal of Development Research*, 6(4) (2016) 7402- 7408.
13. N.Meenakumari and T.Indira, Few applications of $(r^*g^*)^*$ closed sets in topological spaces, *International Journal of Mathematics and computer Research*, 4(5) (2016) 12-32.
14. O.Njasted, On some classes of nearly open sets, *Pacific J. Math*, 15 (1965) 961 - 970.
15. M.Rajamani, Studies on decomposition of generalized continuous maps in topological spaces, Ph.D. Thesis Bharathiar University, Coimbatore, (2001).
16. J.A.Tong, Decomposition of continuity, *Acta Math Hunger*, 48 (1986) 11- 15.
17. J.Tong, Weak almost continuous mapping and weak nearly compact spaces, *Boll. Un. Mat.*, 6 (1982) 385-391.