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Reverse Zagreb and Reverse Hyper-Zagreb Indices and their Polynomials of Rhombus Silicate Networks

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

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Abstract. We propose the first and second reverse hyper-Zagreb indices of a graph. In this paper, we compute the first two reverse Zagreb indices, the first two reverse hyper-Zagreb indices and their polynomials of rhombus silicate networks.

Keywords: reverse Zagreb index, reverse hyper-Zagreb index, rhombus silicate network

AMS Mathematics Subject Classification (2010): 05C05, 05C12, 05C35

1. Introduction

Let $G = (V(G), E(G))$ be a finite, undirected without loops and multiplied edges. The degree $d_G(v)$ is the number of vertices adjacent to v. Let $\Delta(G)$ denote the maximum degree among the vertices of G. The reverse vertex degree of a vertex v in G is defined as $c_v = \Delta(G) - d_G(v)+1$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. We refer [1] for undefined term and notation.

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Chemical graph theory has an important effect on the development of the Chemical Sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Numerous topological indices have been considered in Theoretical Chemistry see [2].

The first reverse Zagreb beta index and second reverse Zagreb index [3] of a graph G are respectively defined as

$$CM_1(G) = \sum_{uv\in E(G)} (c_u + c_v), \qquad CM_2(G) = \sum_{uv\in E(G)} c_u c_v. \tag{1}$$

These indices were also studied, for example, in [4, 5].

We now introduce the first and second reverse hyper-Zagreb indices of a graph G as

$$HCM_1(G) = \sum_{uv\in E(G)} (c_u + c_v)^2, \qquad HCM_2(G) = \sum_{uv\in E(G)} (c_u c_v)^2. \tag{2}$$

Considering the first and second reverse Zagreb indices, we introduce the first and second reverse Zagreb polynomials as

$$CM_1(G,x)=\sum_{uv\in E(G)} x^{c_u+c_v}, \qquad CM_2(G,x)=\sum_{uv\in E(G)} x^{c_u c_v}. \tag{3}$$

Also considering the first and second reverse hyper-Zagreb indices, we introduce the first and second reverse hyper-Zagreb polynomials as

$$HCM_1(G,x)=\sum_{uv\in E(G)} x^{(c_u+c_v)^2}, \qquad HCM_2(G,x)=\sum_{uv\in E(G)} x^{(c_u c_v)^2}, \tag{4}$$

Recently many topological indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14,15,16].

In this paper, we determine the first two reverse Zagreb indices, the first two reverse hyper-Zagreb indices, and their polynomials of rhombus silicate networks. For networks see [17] and references cited therein.

2. Results for Rhombus Silicate networks

Silicates are obtained by fusing metal oxides or metal carbonates with sand. In this section, we consider a family of rhombus silicate networks. This network is symbolized by $RHSL_n$. A 3-dimensional rhombus silicate network is depicted in Figure 1.

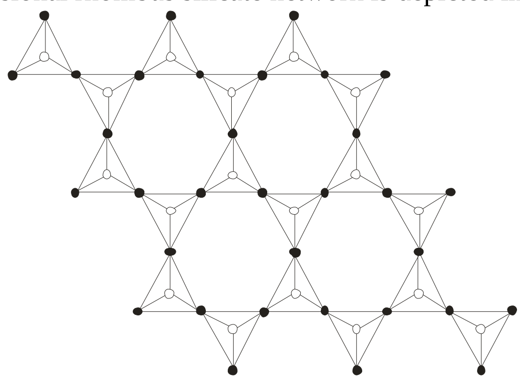

Figure 1: A 3-dimensional rhombus silicate network

Theorem 1. The first and second reverse Zagreb indices of rhombus silicate network $RHSL_n$ are

(i) $CM_1(RHSL_n) = 42n^2+36n$. (ii) $CM_2(RHSL_n) = 30n^2+72n+18$.

Proof: Let G be the graph of rhombus silicate network $RHSL_n$. The graph G has $5n^2 + 2n$ vertices and $12n^2$ edges. From Figure 1, we see that the vertices of $RHSL_n$ are either of degree 3 or 6. Thus $\Delta(G) = 6$. In $RHSL_n$, by algebraic method, there are three types of edges as follows:

$E_{33}= \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 4n+2$.

$E_{36} = \{uv\in E(G) \mid d_G(u) \mid = 3,\ d_G(v) = 6\}$, $|E_{36}| = 6n^2 + 4n - 4$.

$E_{66} = \{uv\in E(G) \mid d_G(u) \mid = d_G(v) = 6\}$, $|E_{66}| = 6n^2 - 8n + 2$.

Clearly we have $c_u = \Delta(G) - d_G(u) + 1 = 7 - d_G(u)$.

We now see that there are three types of reverse edges as follows:

$CE_{44} = \{uv\in E(G) \mid c_u = c_v = 4\}$, $|CE_{44}| = 4n + 2$.

$CE_{41} = \{uv\in E(G) \mid c_u = 4, c_v = 1\}$, $|CE_{41}| = 6n^2 + 4n - 4$.

$CE_{11} = \{uv\in E(G) \mid c_u = c_v = 1\}$, $|CE_{11}| = 6n^2 - 8n + 2$.

(i) To compute $CM_1(RHSL_n)$, we see that

$$CM_1(RHSL_n) = \sum_{uv\in E(G)} (c_u + c_v) = \sum_{EE_{44}} (c_u + c_v) + \sum_{RE_{41}} (c_u + c_v) + \sum_{RE_{11}} (c_u + c_v)$$
$$= (4+4)\,(4n + 2) + (4+1)(6n^2 + 4n - 4) + (1+1)(6n^2 - 8n + 2)$$
$$= 42n^2+36n.$$

(ii) To compute $CM_2(RHSL_n)$, we see that

$$CM_2(RHSL_n) = \sum_{uv\in E(G)} c_u c_v = \sum_{EE_{44}} c_u c_v + \sum_{RE_{41}} c_u c_v + \sum_{RE_{11}} c_u c_v$$
$$= (4\times4)\,(4n + 2) + (4\times1)(6n^2 + 4n - 4) + (1\times1)(6n^2 - 8n + 2)$$
$$= 30n^2 + 72n + 18.$$

Theorem 2. The first and second reverse Zagreb polynomials of rhombus silicate network $RHSL_n$ are

(i) $CM_1(RHSL_n, x) = (4n+2)\, x^8 + (6n^2 + 4n - 4)\, x^5 + (6n^2 - 8n + 2)\, x^2$.

(ii) $CM_2(RHSL_n, x) = (4n+2)\, x^{16} + (6n^2 + 4n - 4)\, x^4 + (6n^2 - 8n + 2)\, x$.

Proof: Let $G= RHSL_n$

(i) From equation (3) and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$CM_1(RHSL_n, x) = \sum_{uv\in E(G)} x^{c_u+c_v} = \sum_{CE_{44}} x^{c_u+c_v} + \sum_{CE_{41}} x^{c_u+c_v} + \sum_{CE_{11}} x^{c_u+c_v}$$
$$= (4n+2)\,x^{4+4} + (6n^2+4n-4)\,x^{4+1} + (6n^2-8n+2)\,x^{1+1}$$
$$= (4n+2)\,x^{8} + (6n^2+4n-4)\,x^{5} + (6n^2-8n+2)\,x^{2}.$$

(ii) From equation (3), and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$CM_2(RHSL_n, x) = \sum_{uv\in E(G)} x^{c_u c_v} = \sum_{CE_{44}} x^{c_u c_v} + \sum_{CE_{41}} x^{c_u c_v} + \sum_{CE_{11}} x^{c_u c_v}$$
$$= (4n+2)\,x^{4\times4} + (6n^2+4n-4)\,x^{4\times1} + (6n^2-8n+2)\,x^{1\times1}$$
$$= (4n+2)\,x^{16} + (6n^2+4n-4)\,x^{4} + (6n^2-8n+2)\,x.$$

Theorem 3. The first and second reverse hyper-Zagreb indices of rhombus silicate network $RHSL_n$ are

(i) $HCM_1(RHSL_n) = 174n^2 + 324n + 36$.

(ii) $HCM_2(RHSL_n) = 102n^2 + 1080n + 450$.

Proof: Let $G = RHSL_n$.

(i) From equation (2) and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$HCM_1(RHSL_n) = \sum_{uv\in E(G)} (c_u + c_v)^2 = \sum_{CE_{44}} (c_u + c_v)^2 + \sum_{CE_{41}} (c_u + c_v)^2 + \sum_{CE_{11}} (c_u + c_v)^2$$
$$= (4+4)^2 (4n+2) + (4+1)^2 (6n^2+6n-4) + (1+1)^2 (6n^2-8n+2)$$
$$= 174n^2 + 324n + 36.$$

(ii) From equation (2) and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$HCM_2(RHSL_n) = \sum_{uv\in E(G)} (c_u c_v)^2 = \sum_{CE_{44}} (c_u c_v)^2 + \sum_{CE_{41}} (c_u c_v)^2 + \sum_{CE_{11}} (c_u c_v)^2$$

$$= (4\times4)^2 (4n+2) + (4\times1)^2 \left(6n^2+6n-4\right) + (1\times1)^2 \left(6n^2-8n+2\right)$$

$$= 102n^2 + 1080n + 450.$$

Theorem 4. The first and second reverse hyper-Zagreb polynomials of rhombus silicate network $RHSL_n$ are

(i) $HCM_1(RHSL_n, x) = (4n+2)x^{64} + (6n^2 + 4n - 2)x^{25} + (6n^2 - 8n + 2)x^4$.

(ii) $HCM_2(RHSL_n, x) = (4n+2)x^{256} + (6n^2 + 4n - 2)x^{16} + (6n^2 - 8n + 2)x$.

Proof: Let $G = RHSL_n$.

(i) From equation (4) and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$HCM_1(RHSL_n, x) = \sum_{uv\in E(G)} x^{(c_u+c_v)^2} = \sum_{CE_{44}} x^{(c_u+c_v)^2} + \sum_{CE_{41}} x^{(c_u+c_v)^2} + \sum_{CE_{11}} x^{(c_u+c_v)^2}$$

$$= (4n+2)x^{(4+4)^2} + \left(6n^2+6n-4\right)x^{(4+1)^2} + \left(6n^2-8n+2\right)x^{(1+1)^2}$$

$$= (4n+2)x^{64} + \left(6n^2+6n-4\right)x^{25} + \left(6n^2-8n+2\right)x^4.$$

(ii) From equation (4) and by cardinalities of the reverse edge partition of $RHSL_n$, we have

$$HCM_2(RHSL_n, x) = \sum_{uv\in E(G)} x^{(c_u c_v)^2} = \sum_{CE_{44}} x^{(c_u c_v)^2} + \sum_{CE_{41}} x^{(c_u c_v)^2} + \sum_{CE_{11}} x^{(c_u c_v)^2}$$

$$= (4n+2)x^{(4\times4)^2} + \left(6n^2+6n-4\right)x^{(4\times1)^2} + \left(6n^2-8n+2\right)x^{(1\times1)^2}$$

$$= (4n+2)x^{256} + \left(6n^2+6n-4\right)x^{16} + \left(6n^2-8n+2\right)x.$$

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