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One Modulo Three Root Square Mean Labeling of Some Disconnected Graphs

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Abstract. A graph G is said to be one modulo three root square mean graph if there is an injective function φ from the vertex set of G to the set $\{0, 1, 3, \ldots, 3q-2, 3q\}$ where q is the number of edges of G and φ induces a bijection φ* from the edge set of G to $\{1, 4, \ldots, 3q-2\}$ given by $\varphi^*(uv) = \left\lceil\sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}}\right\rfloor$ and the function φ is called one modulo three root square mean labeling of G. The concept of one modulo three root square mean labeling was introduced by Jayasekaran and Jaslin Melbha and they investigated some graphs are one modulo three root square mean graphs. In this paper we prove that some disconnected graphs are one modulo three root square mean labeling.

Keywords: one modulo three root square mean labeling, one modulo three root square mean graphs.

AMS Mathematics Subject Classification (2010)*:* 05C99, 05C22

1. Introduction

We begin with simple, finite, connected and undirected graph. For standard terminology and notations we follow Harary [1]. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (an *edge labeling*). Several types of graph labeling and a detailed survey are available in [2]. A very good application of graph labeling is given in [3,4]

Jayasekaran and David Raj introduced the concept one modulo three harmonic mean labeling of graphs in [5]. Root square mean labeling was introduced by Sandhya, Somasundaram and Anusa in [6]. Jayasekaran and Jaslin Melbha introduced the concept one modulo three root square mean labeling of graphs in [7]. Further they investigated some graphs are one modulo three root square mean graphs [8]. Not every graph is one modulo three root square mean. For example, star graph $K_{1, n}$, where $n \geq 4$ is not a one modulo three root square mean graph. We are interested to study different classes of graphs, which are one modulo three root square mean graphs.

We will provide a brief summary of definitions and other information's which are necessary for our present investigation.

Definition 1.1. A graph G is said to be one modulo three root square mean graph if there is an injective function φ from the vertex set of G to the set {0, 1, 3, …, $3q$-2, $3q$} where q is the number of edges of G and φ induces a bijection φ* from the edge set of G to{1, 4, …, $3q$-2} given by $\varphi^*(uv) = \left\lceil \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rfloor$ and the function φ is called one modulo three root square mean labeling of G.

Definition 1.2. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \Theta G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where i^{th} vertex of G_1 is adjacent to every vertices in the i^{th} copy of G_2.

Definition 1.3. The graph $P_n \Theta K_1$ is called a *comb*.

Definition 1.4. The product $P_2 \times P_n$ is called a *ladder* and it is denoted by L_n. The ladder graph L_n is a planar undirected graph with 2n vertices and 3n-2 edges.

Definition 1.5. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

Theorem 1.6. The ladder L_n is a one modulo three root square mean graph for $n \neq 2$ [6].

Theorem 1.7. $P_m \cup (P_n \Theta K_1)$ is a one modulo three root square mean graph [6].

2. Main results

Theorem 2.1. $(L_m \Theta K_1) \cup P_n$ is one modulo three root square mean graph.

Proof: Let $u_1u_2\ldots u_m$ and $v_1v_2\ldots v_m$ be two paths of length m. Join u_i and v_i, $1 \le i \le m$. The resultant graph is L_m. For $1 \le i \le m$, let x_i be the pendant vertex adjacent to u_i and y_i be the pendant vertex adjacent to v_i. Then we get the graph $L_m \Theta K_1$. Let $w_1w_2\ldots w_n$ be the path P_n. Let $G = (L_m \Theta K_1) \cup P_n$ with $V(G) = \{u_i, v_i, x_i, y_i, w_j / 1 \le i \le m, 1 \le j \le n\}$ and $E(G) = \{u_iv_i, u_ix_i, v_iy_i, v_jv_{j+1}, u_ju_{j+1}, w_jw_{j+1} / 1 \le i \le m, 1 \le j \le n-1\}$. Then G has 4m+n vertices and 5m+n-3 edges. Define a function $\phi : V(G) \to \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 1$, $\phi(u_2) = 15$, $\phi(u_3) = 33$, $\phi(u_4) = 46$, $\phi(u_i) = 15i-12$, $5 \le i \le m$; $\phi(x_1) = 0$, $\phi(x_2) = 22$, $\phi(x_3) = 28$, $\phi(x_4) = 45$, $\phi(x_i) = 15i-17$, $5 \le i \le m$; $\phi(v_1) = 6$, $\phi(v_2) = 18$, $\phi(v_3) = 36$, $\phi(v_i) = 15i-9$, $4 \le i \le m$; $\phi(y_1) = 9$, $\phi(y_2) = 24$, $\phi(y_3) = 39$, $\phi(y_i) = 15i-8$, $4 \le i \le m$; $\phi(w_i) = 15m+3i-9$, $1 \le i \le n$. Then ϕ induces a bijection $\phi^*: E(G) \to \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 15i-5$, $1 \le i \le m-1$; $\phi^*(u_1x_1) = 1$, $\phi^*(u_2x_2) = 19$, $\phi^*(u_ix_i) = 15i-14$, $3 \le i \le m$; $\phi^*(u_1v_1) = 4$, $\phi^*(u_2v_2) = 16$, $\phi^*(u_iv_i) = 15i-11$, $3 \le i \le m$; $\phi^*(v_1y_1) = 7$, $\phi^*(v_2y_2) = 22$, $\phi^*(v_iy_i) = 15i-8$, $3 \le i \le m$; $\phi^*(v_iv_{i+1}) = 15i-2$, $1 \le i \le m-1$; $\phi^*(w_iw_{i+1}) = 15m+3i-8$, $1 \le i \le n-1$. Hence $(L_m \Theta K_1) \cup P_n$ is one modulo three root square mean graph.

Example 2.2. A one modulo three root square mean labeling of $(L_8 \Theta K_1) \cup P_7$ is given in figure 1.

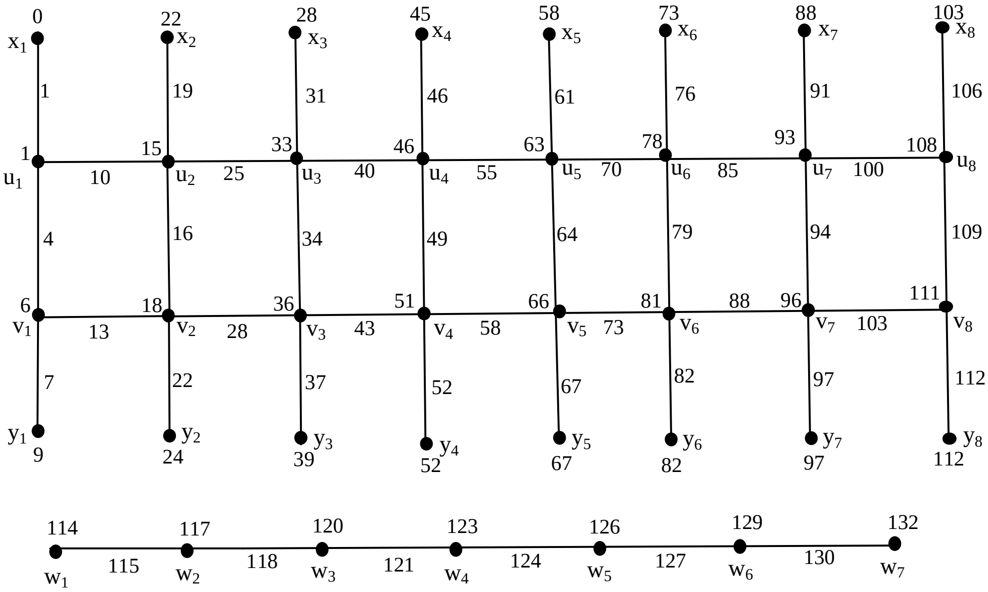

Figure 1: $(L_8 \Theta K_1) \cup P_7$

Theorem 2.3. $(L_m \Theta K_1) \cup (P_n \Theta K_1)$ is one modulo three root square mean graph.

Proof: Let $u_1u_2\ldots u_m$ and $v_1v_2\ldots v_m$ be two paths of length m. Join u_i and v_i, $1\le i\le m$. The resultant graph is L_m. For $1\le i\le m$, let x_i be the pendant vertex adjacent to u_i and y_i be the pendant vertex adjacent to v_i. Then we get the graph $L_m \Theta K_1$. Let P_n be the path $v_1v_2\ldots v_n$. Let w_i be the vertex adjacent to v_i, $1\le i\le n$. The resultant graph is $P_n\Theta K_1$. Let $G = (L_m \Theta K_1) \cup (P_n\Theta K_1)$ with $V(G) = \{u_i, v_i, x_i, y_i, s_j, t_j / 1\le i\le m, 1\le j\le n\}$ and $E(G) = \{u_iv_i, u_ix_i, v_iy_i, v_jv_{j+1}, u_ju_{j+1}, s_jt_j, s_js_{j+1}/1\le i\le m, 1\le j\le n\text{-}1\}$. Then G has 4m+2n vertices and 5m+2n-3 edges. Define a function $\phi : V(G)\rightarrow \{0, 1, 3,\ldots, 3q\text{-}2, 3q\}$ by $\phi(u_1) = 1$, $\phi(u_2) = 15$, $\phi(u_3) = 33$, $\phi(u_4) = 46$, $\phi(u_i) = 15i\text{-}12$, $5 \le i\le m$; $\phi(x_1)= 0$, $\phi(x_2)= 22$, $\phi(x_3)= 28$, $\phi(x_4)= 45$, $\phi(x_i)= 15i\text{-}17$, $5 \le i\le m$; $\phi(v_1)= 6$, $\phi(v_2) = 18$, $\phi(v_3) = 34$, $\phi(v_i) = 15i\text{-}9$, $4 \le i\le m$; $\phi(y_1) = 9$, $\phi(y_2) = 24$, $\phi(y_3) = 39$, $\phi(y_i) = 15i\text{-}8$, $4\le i\le m$; $\phi(s_i) = 15m+6i\text{-}9$ for all odd $i \le n$; $\phi(s_i) =15m+6i\text{-}12$ for all even $i \le n$; $\phi(t_i) = 15m+6i\text{-}12$ for all odd $i \le n$; $\phi(t_i) = 15m+6i\text{-}9$ for all even $i \le n$. Then ϕ induces a bijection $\phi^*: E(G)\rightarrow \{1, 4, \ldots, 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 15i\text{-}5$, $1\le i \le m\text{-}1$; $\phi^*(u_1x_1) = 1$, $\phi^*(u_2x_2) = 19$, $\phi^*(u_ix_i) = 15i\text{-}14$, $3 \le i \le m$; $\phi^*(u_1v_1) = 4$, $\phi^*(u_2v_2) = 16$, $\phi^*(u_iv_i) = 15i\text{-}11$, $3 \le i \le m$; $\phi^*(v_1y_1) = 7$, $\phi^*(v_2y_2) = 22$, $\phi^*(v_iy_i) = 15i\text{-}8$, $3 \le i \le m$; $\phi^*(v_iv_{i+1}) = 15i\text{-}2$, $1\le i \le m\text{-}1$; $\phi^*(s_it_i) = 15m+6i\text{-}11$, $1\le i \le n$; $\phi^*(s_is_{i+1}) = 15m+6i\text{-}8$, $1\le i \le n\text{-}1$. Hence $(L_m \Theta K_1) \cup (P_n\Theta K_1)$ is one modulo three root square mean graph.

Example 2.4. A one modulo three root square mean labeling of $(L_8\Theta K_1) \cup (P_6\Theta K_1)$ is given in figure 2.

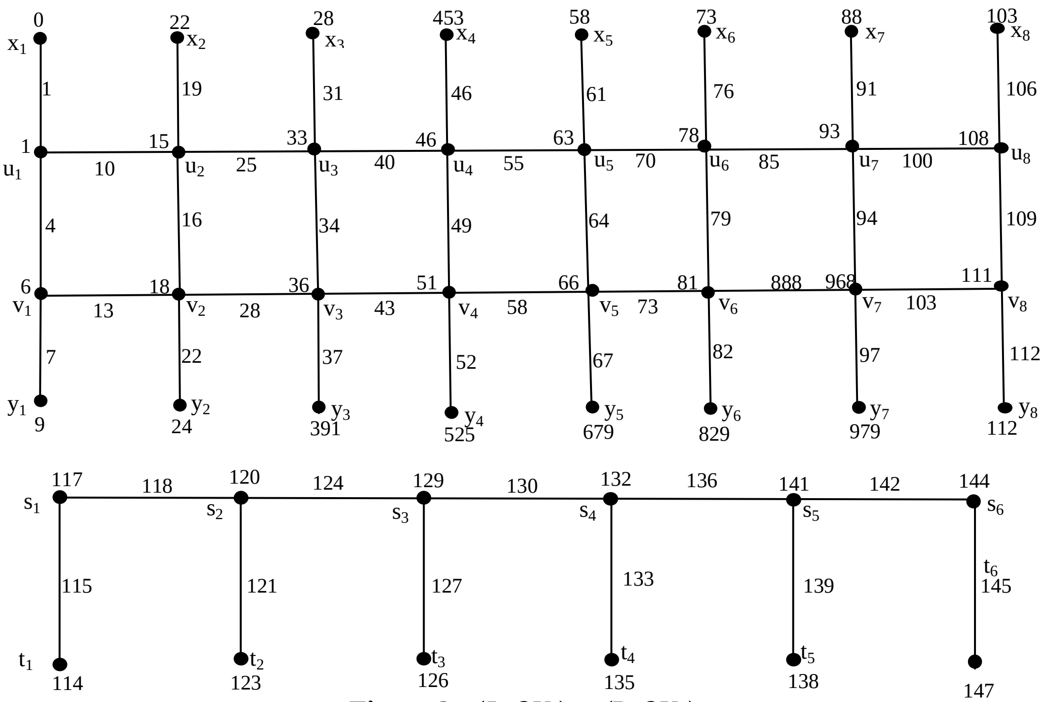

Figure 2: $(L_8 ʘ K_1) \cup (P_6 ʘ K_1)$

Theorem 2.5. $P_n \cup P_m$ is a one modulo three root square mean graph.

Proof: Let $u_1u_2\ldots u_n$ be the path P_n and $v_1v_2\ldots v_m$ be the path P_m. Let $G = P_n \cup P_m$ with $V(G) = \{u_i, v_j /1\leq i\leq n, 1\leq j\leq m\}$ and $E(G) = \{u_iu_{i+1}, v_jv_{j+1} /1\leq i\leq n-1, 1\leq j\leq m-1\}$. Then G has n+m vertices and n+m-2 edges. Define a function $\phi : V(G) \to \{0, 1, 3,\ldots, 3q-2, 3q\}$ by $\phi(u_1) = 0$, $\phi(u_2) = 1$, $\phi(u_i) = 3(i-1)$, $3 \leq i \leq n-1$, $\phi(u_n) = 3n-5$; $\phi(v_j) = 3n+3j-6$, $1\leq j \leq m-1$, $\phi(v_m) = 3n+3j-8$. Then ϕ induces a bijection $\phi^*: E(G) \to \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 3i-2$, $1\leq i \leq n-1$; $\phi^*(v_jv_{j+1}) = 3n+3j-5$, $1\leq j\leq m-1$. Hence $P_n \cup P_m$ is a one modulo three root square mean graph .

Example 2.6. A one modulo three root square mean labeling of $P_7 \cup P_6$ is given in figure 3.

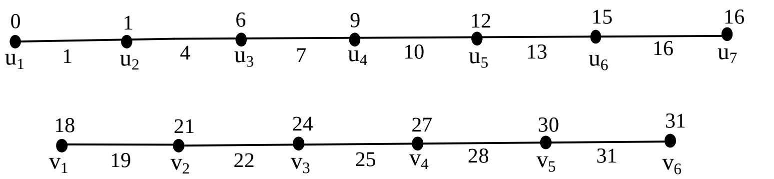

Figure 3: $P_7 \cup P_6$

Theorem 2.7. $L_m \cup P_n$ is a one modulo three root square mean graph for $m \neq 2$ and $n \neq 3$.

Proof: Let $u_1u_2\ldots u_m$ and $v_1v_2\ldots v_m$ be two paths of length m. Join u_i and v_i, $1\leq i\leq m$. The resultant graph is L_m. Let $w_1w_2\ldots w_n$ be the path P_n. Let $G = L_m \cup P_n$ with $V(G) = \{u_i, v_i, w_j /1\leq i\leq m, 1\leq j\leq n\}$ and $E(G) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_i, u_nv_n, w_jw_{j+1}/1\leq i\leq m-1, 1\leq j\leq n-1\}$. Then G has 2m+n vertices and 3m+n-3 edges.

Case 1. m = 1

In this case, $L_1 \cup P_n = P_2 \cup P_n$. By Theorem 2.5 $P_2 \cup P_n$ is a one modulo three root square mean graph.

Case 2. $m = 2$

Sub case 2.1. $n = 2$

In this case, $L_m \cup P_n = L_2 \cup P_2$. Define a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 3$, $\phi(u_2) = 4$, $\phi(v_1) = 15$, $\phi(v_2) = 10$, $\phi(w_1) = 0$, $\phi(w_2) = 1$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 4$, $\phi^*(u_1v_1) = 10$, $\phi^*(u_2v_2) = 7$, $\phi^*(v_1v_2) =13$, $\phi^*(w_1w_2) =1$. Thus the edges get distinct labels 1, 4, …, 13. In this case ϕ is a one modulo three root square mean labeling for G. A one modulo three root square mean labeling of $L_2 \cup P_2$ is given in figure 4.

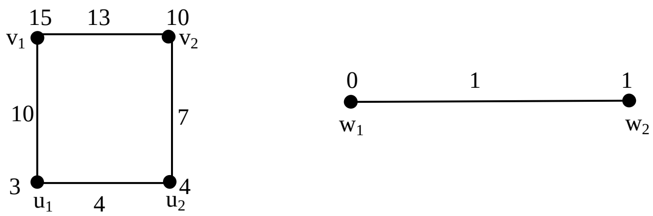

Figure 4: $L_2 \cup P_2$

Sub case 2.2. $n = 3$

In this case, $L_m \cup P_n = L_2 \cup P_3$. It has 6 edges and 7 vertices. When we label these 7 vertices from the vertex set {0, 1, 3, …, 18}, atleast two edges get the same labels and hence $L_2 \cup P_3$ is not one modulo three root square mean labeling.

Sub case 2.3. $n \geq 4$

In this case, $L_m \cup P_n = L_2 \cup P_n$. Define a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 3n+9$, $\phi(u_2) = 3n+4$; $\phi(v_1) = 3n-3$, $\phi(v_2) = 3n-2$; $\phi(w_1) = 0$, $\phi(w_2) = 1$, $\phi(w_i) = 3(i-1)$, $4 \leq i \leq n-1$; $\phi(w_n) = 3n-5$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 3n+7$, $\phi^*(u_2v_2) = 3n+1$, $\phi^*(u_1v_1) = 3n+4$, $\phi^*(v_1v_2) = 3n-2$; $\phi^*(w_iw_{i+1})= 3i-2$, $1\leq i\leq n-1$. Thus the edges get distinct labels 1, 4, …, 3q-2. In this case ϕ is a one modulo three root square mean labeling for G.

Case 3. $m \geq 3$

Define a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 0$, $\phi(u_2) = 1$, $\phi(u_3) = 6$, $\phi(u_i) = 9i-6$, $4\leq i\leq m$; $\phi(v_i) = 6i+3$, $1\leq i\leq 3$; $\phi(v_i) = 9i-9$, $4\leq i\leq m$; $\phi(w_1) = 9m-8$, $\phi(w_i) = 9(m-1)+3i$, $2\leq i \leq n$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 1$, $\phi^*(u_2u_3)= 4$, $\phi^*(u_3u_4)= 22$, $\phi^*(u_iu_{i+1})= 9i-2$, $4\leq i \leq m-1$; $\phi^*(v_iv_{i+1})= 6i+7$, $1\leq i \leq 3$; $\phi^*(v_iv_{i+1})= 9i-5$, $4\leq i\leq m-1$; $\phi^*(u_1v_1)= 7$, $\phi^*(u_2v_2) = 10$, $\phi^*(u_3v_3) =16$, $\phi^*(u_iv_i) = 9i-8$, $4\leq i\leq m$; $\phi^*(w_iw_{i+1}) = 9m+3i-8$, $1\leq i\leq n-1$. Thus the edges get distinct labels 1, 4, …, 3q-2. In this case ϕ is a one modulo three root square mean labeling for G. Hence $L_m \cup P_n$ is a one modulo three root square mean graph for $m \neq 2$ and $n \neq 3$.

Example 2.8. A one modulo three root square mean labeling of $L_2 \cup P_4$ and $L_9 \cup P_7$ are shown in figure 5 and figure 6 respectively.

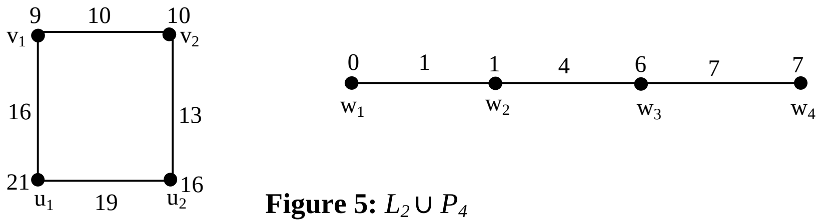

Figure 5: $L_2 \cup P_4$

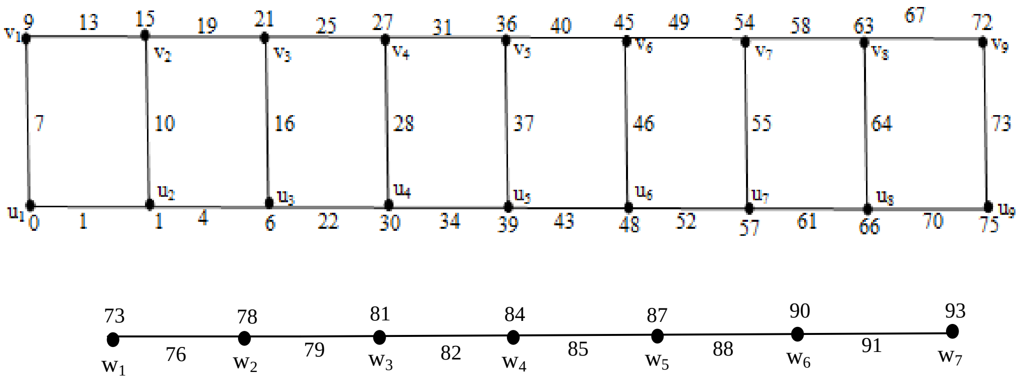

Figure 6: $L_9 \cup P_7$

Theorem 2.9. $L_m \cup (P_n \odot K_1)$ is a one modulo three root square mean graph.

Proof: Let $u_1u_2\ldots u_m$ and $v_1v_2\ldots v_m$ be two paths of length m. Join u_i and v_i, $1 \leq i \leq m$. The resultant graph is L_m. Let $s_1s_2\ldots s_n$ be the path P_n. For $1 \leq i \leq n$, join t_i with s_i. The resultant graph is $P_n \odot K_1$. Let $G = L_m \cup (P_n \odot K_1)$ with $V(G) = \{u_i, v_i, s_j, t_j \,/ 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_i, u_mv_m, s_js_{j+1}, s_jt_j, s_nt_n \,/ 1 \leq i \leq m-1, 1 \leq j \leq n-1\}$. Then G has 2m+2n vertices and 3m+2n-3edges.

Case 1. m = 1

In this case, $L_1 \cup (P_n \odot K_1) = P_2 \cup (P_n \odot K_1)$. By Theorem 1.7 $P_m \cup (P_n \odot K_1)$ is a one modulo three root square mean graph.

Case 2. m = 2

Sub case 2.1. $n = 1$

In this case, $L_m \cup (P_n \odot K_1) = L_2 \cup P_2$. By Theorem 2.7, $L_2 \cup P_2$ is a one modulo three root square mean graph.

Sub case 2.2. $n = 2$

In this case, $L_m \cup (P_n \odot K_1) = L_2 \cup P_4$. By Theorem 2.7, $L_2 \cup P_4$ is a one modulo three root square mean graph.

Sub case 2.3. $n \geq 3$

In this case, $L_m \cup (P_n \odot K_1) = L_2 \cup (P_n \odot K_1)$. Define a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 6n+9$, $\phi(u_2) = 6n+4$; $\phi(v_1) = 6n-3$, $\phi(v_2) = 6n-2$; $\phi(s_1) = 0$, $\phi(s_i) = 6i-5$, $2 \leq i \leq n$; $\phi(t_1) = 1$, $\phi(t_i) = 6i-6$, $2 \leq i \leq n$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 6n+7$, $\phi^*(u_2v_2) = 6n+1$, $\phi^*(u_1v_1) = 6n+4$, $\phi^*(v_1v_2) = 6n-2$; $\phi^*(s_is_{i+1}) = 6i-2$, $1\leq i\leq n-1$; $\phi^*(s_it_i) = 6i-5$, $1\leq i\leq n$. Thus the edges get distinct labels $\{1, 4, \ldots, 3q-2\}$. In this case ϕ is a one modulo three root square mean labeling for G.

Case 3. $m \geq 3$

Define a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 0$, $\phi(u_2) = 1$, $\phi(u_3) = 6$, $\phi(u_i)= 9i-6$, $4\leq i\leq m$; $\phi(v_i) = 6i+3$, $1\leq i\leq 3$; $\phi(v_i) = 9(i-1)$, $4\leq i \leq m$; $\phi(t_1) = 9m-8$, $\phi(t_i) = 9m+6i-12$ for all odd i, $3\leq i\leq n$; $\phi(t_i) = 9m+6i-9$ for all even $i \leq n$; $\phi(s_i) = 9m+6i-9$ for all odd $i \leq n$; $\phi(s_i) = 9m+6i-12$ for all even $i \leq n$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 1$, $\phi^*(u_2u_3) = 4$, $\phi^*(u_3u_4) = 22$, $\phi^*(u_iu_{i+1}) = 9i-2$, $4\leq i\leq m-1$; $\phi^*(v_iv_{i+1}) = 6i+7$, $1\leq i \leq 3$; $\phi^*(v_iv_{i+1})= 9i-5$, $4\leq i \leq m-1$; $\phi^*(u_1v_1) = 7$, $\phi^*(u_2v_2) = 10$, $\phi^*(u_3v_3) = 16$, $\phi^*(u_iv_i) = 9i-8$, $4\leq i \leq m$; $\phi^*(s_it_i)= 9m+6i-11$, $1\leq i\leq n$; $\phi^*(s_is_{i+1})= 9m+6i-8$, $1\leq i\leq n-1$. Thus the edges get distinct labels $1, 4, \ldots, 3q-2$. In this case ϕ is a one modulo three root square mean labeling for G. Hence $L_m \cup (P_n \odot K_1)$ is a one modulo three root square mean graph.

Example 2.10. A one modulo three root square mean labeling of $L_2 \cup (P_4 \odot K_1)$ and $L_7 \cup (P_6 \odot K_1)$ are shown in figure 7 and figure 8 respectively.

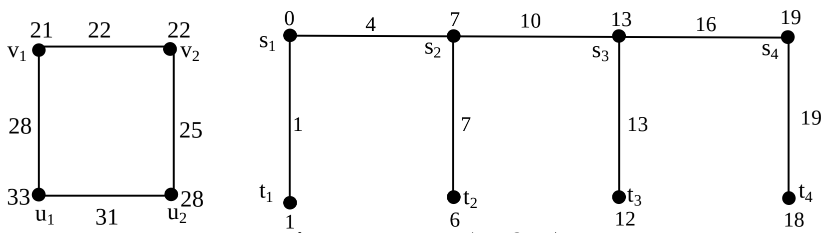

Figure 7: $L_2 \cup (P_4 \odot K_1)$

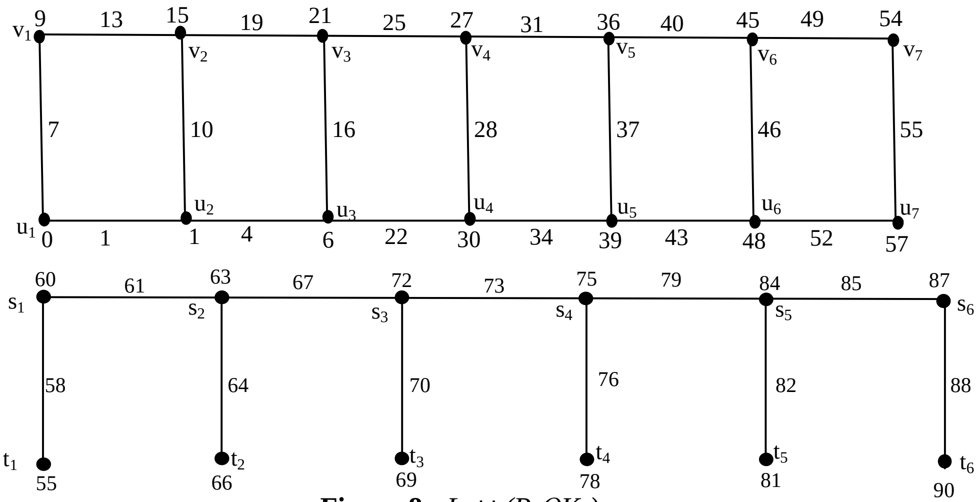

Figure 8: $L_7 \cup (P_6 \odot K_1)$

Theorem 2.11. $(P_m \odot \overline{K}_3) \cup P_n$ is a one modulo three root square mean graph.

Proof: Let $u_1u_2 \ldots u_m$ be the path P_m. Let v_i, x_i, y_i, z_i be the vertices of i^{th} copy of $K_{1,3}$ with central vertex v_i. Identify v_i with u_i, $1 \le i \le m$. Let $w_1w_2 \ldots w_n$ be the path P_n. The resultant graph is $G = (P_m \odot \overline{K}_3) \cup P_n$ with $V(G) = \{u_i, x_i, y_i, z_i, w_j / 1 \le i \le m, 1 \le j \le n\}$ and $E(G) = \{u_ix_i, u_iy_i, u_iz_i, u_ju_{j+1}, w_jw_{j+1} / 1 \le i \le m, 1 \le j \le n-1\}$. Then G has 4m+n vertices and 4m+n-2 edges. Define a function $\phi: V(G) \rightarrow \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 1$; $\phi(u_2) = 15$; $\phi(u_i) = 12i-8$, $3 \le i \le m$; $\phi(x_1) = 0$; $\phi(x_i) = 12i-14$, $2 \le i \le m$; $\phi(y_1) = 6$; $\phi(y_2) = 16$; $\phi(y_i) = 12i-9$, $3 \le i \le m$; $\phi(z_i) = 12i-3$, $1 \le i \le m$; $\phi(w_1) = 12m-5$, $\phi(w_i) = 12m+3i-6$, $2 \le i \le n$. Then ϕ induces a bijection $\phi^*: E(G) \rightarrow \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 12i-2$, $1 \le i \le m-1$; $\phi^*(u_ix_i) = 12i-11$, $1 \le i \le m$; $\phi^*(u_iy_i) = 12i-8$, $1 \le i \le m$; $\phi^*(u_iz_i) = 12i-5$, $1 \le i \le m$; $\phi^*(w_iw_{i+1}) = 12m+3i-5$, $1 \le i \le n-1$. Therefore, ϕ is a one modulo three root square mean labeling. Hence $(P_m \odot \overline{K}_3) \cup P_n$ is a one modulo three root square mean graph.

Example 2.12. One modulo three root square mean labeling of $(P_5 \odot \overline{K}_3) \cup P_7$ is given in figure 6.

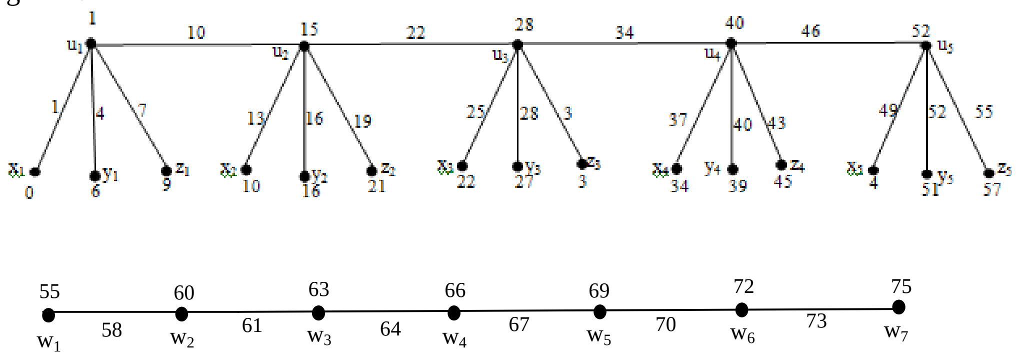

Figure 9: $(P_5 \odot \overline{K}_3) \cup P_7$

3. Conclusion

In this paper, we prove that some disconnected graphs $(L_m \odot K_1) \cup P_n$, $(L_m \odot K_1) \cup (P_n \odot K_1)$, $P_n \cup P_m$, $L_m \cup (P_n \odot K_1)$, $(P_m \odot \overline{K}_3) \cup P_n$ and $L_m \cup P_n$, for $m \neq 2$ and $n \neq 3$ are one modulo three root square mean labeling.

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