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Some Star Related Vertex Odd Divisor Cordial Labeling of Graphs

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Abstract. In this paper we prove that the graphs

$D_2(K_{1,n})$, $K_{2,n} \odot u_2(K_1)$ $\prec K_{1,n} \blacktriangle K_{1,n} \succ$, $\prec B_{n,n} : w \succ$ and $G = K_{1,n} * P_{n+2}$,

are vertex odd divisor cordial graphs. Also, investigated lots of properties.

Keywords: labeling, cordial labeling, divisor cordial labeling, vertex odd divisor cordial labeling.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graph theory has several interesting applications in system analysis, operations research and economics. Since most of the time the aspects of graph problems are uncertain, it is nice to deal with these aspects via the methods of labeling. The concept of labeling of graphs is an active research area and it has been widely studied by several researchers. In a wide area network (WAN), several systems are connected to the main server, the labeling technique plays a vital role to label the cables. The labeling of graphs has been applied in the fields such as circuit design, communication network, coding theory, and crystallography.

A graph labeling is a process in which each vertex is assigned a value from the given set of numbers, the labeling of edges depends on the labels of its end vertices. An excellent survey of various graph labeling problems, we refer to Gallian [2]. Two well-known graph labeling methods are graceful labeling and harmonious labeling. These labelings are studied by Cahit [1]. Graph colouring is an active research area in graph theory. Saha et al. [5] have discussed one nice application of graph colouring.

Cordial labeling was introduced by Cahit [1]. Many labeling schemes were introduced with slight variations in cordial such as prime cordial labeling, divisor cordial labeling. Varatharajan et al. [11] have analyzed the divisor cordial labeling.The divisor cordial labeling of various types of the graph is presented in [6,7,8,9,12]. Muthaiyan et al. [4] have introduced the concept of vertex odd divisor cordial graph.

In this paper, we summarize the necessary definitions and basic results in section 2. In section 3, we proved that some standard graphs are vertex odd divisor cordial graph.

We conclude in section 4.

2. Basic definitions

In this section, we provide a brief summary of the definitions and other results which are prerequisites for the present work.

All the graphs considered here are simple finite, undirected without loops and multiple edges. Let $G=(V,E)$ be a graph and as usual, we denote $p=|V|$ and $q=|E|$. For terminology and notations not specifically defined here, we refer to Harary [3].We recall the following definition from Harary [3].

Definition 2.1. Let $G=(V,E)$ be a graph. A mapping $f:V\to\{0,1\}$ is called the binary vertex labeling of G and $f(v)$ is called the label of the vertex $v\in V$ of G under f. The induced edge labeling $f^*:E\to\{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$, for all $e=uv\in E$.

We denote $v_f(i)$ is the number of vertices of G having a label i under f and $e_f(i)$ is the number of edges of G having a label i under f, where $i=0,1$. Now we define cordial labeling of a graph.

Definition 2.2. [1] Let $G=(V,E)$ be a graph and $f:V\to\{0,1\}$ be a binary vertex labeling of G. The map f is called a cordial labeling if $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

A graph G is called cordial graph if it admits cordial labeling.

Definition 2.3. [11] A divisor cordial labeling of a graph $G=(V,E)$ is a bijection $f:V\to\{1,2,3,...,|V|\}$ such that if each edge uv is assigned the label 1 if $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u)\not/ f(v)$, then $|e_f(0)-e_f(1)|\le 1$. A graph which admits divisor cordial labeling is called a vertex divisor cordial graph.

Definition 2.4. [4] A vertex odd divisor cordial labeling of a graph $G=(V,E)$ is a bijection $f:V\to\{1,2,3,...,2n-1\}$ such that if each edge uv is assigned the label 1 if $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u)\,f(v)$, then $|e_f(0)-e_f(1)|\le 1$.

A graph which admits odd divisor cordial labeling is called a vertex odd divisor cordial graph.

Definition 2.5. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex in to the neighbors of the corresponding vertex v' in G''.

Definition 2.6. [12] Consider the graph bistar $B_{m,n}$. Let u,v be the central vertices. We subdivide the edge uv as a path of length 2 by adding a new vertex w is called the subdivision of the bistar. The subdivision of the bistar $B_{m,n}$ is denoted by $\prec B_{m,n} : w \succ$.

Definition 2.7. The graph $K_{1,n} * P_{n+2}$ is a graph obtained from the initial vertex of the path P_{n+2} is attached with the apex vertex of $K_{1,n}$.

Definition 2.8. [12] Let (X, Y) be the bipartition of $K_{m,n}$, where $X = \{u_1, u_2, ..., u_m\}$ and $Y = \{v_1, v_2, ..., v_n\}$. The graph $K_{m,n} \odot u_2(K_1)$ is defined by attaching a pendant vertex to the vertex u_i for some i.

Definition 2.9. [10] Consider two copies of graph G namely G_1 and G_2. Then the graph $G' = \prec G_1 \blacktriangle G_2 \succ$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex v'.

3. Main results

Theorem 3.1. $D_2(K_{1,n})$ is a vertex odd divisor cordial graph.

Proof: Consider the two copies of $K_{1,n}$. Let $v_1, v_2, v_3 .... v_n$ be the pendant vertices of the first copy of $K_{1,n}$ and $u_1, u_2, u_3, ..., u_n$ be the pendant vertices of a second copy of $K_{1,n}$ with u and v are respective apex vertices. Let $G = D_2(K_{1,n})$. Then $|V(G)| = 2n+2$, and $|E(G)| = 4n$ .We define $f : V(G) \to \{1,3,5,...,4n+3\}$ as follows. $f(v) = 1, f(v_i) = 2i+1, 1 \le i \le n, f(u) = p$ where p is the largest prime number such that $2n+3 \le p \le 4n+3$. The remaining vertices $u_i (1 \le i \le n)$ are assigned the labels $2n+3, ..., 4n+3$ in any order except p. From the above labeling, we obtain $e_f(0) = e_f(1) = 2n$.

Thus we have $|e_f(0) - e_f(1)| \le 1$. Hence G is a vertex odd divisor cordial graph.

Example 3.2. Vertex odd divisor cordial labeling of the graph $D_2(K_{1,7})$ is shown in Figure 1.

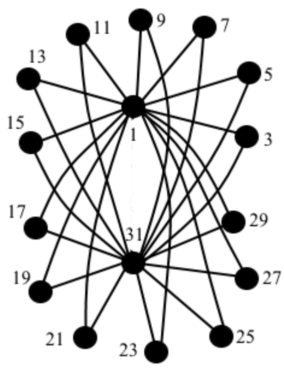

Figure 1: Vertex odd divisor cordial labeling of the graph $D_2(K_{1,7})$

Theorem 3.3. $G = \prec B_{n,n} : w \succ$ is a vertex odd divisor cordial graph.

Proof: Let $u_1, u_2, u_3, ..., u_n$ be the pendant vertices attached to the vertex u and $v_1, v_2, v_3, ..., v_n$ be the pendant vertices attached to the vertex v.

Let $G = \prec B_{n,n} : w \succ$. Then $|V(G)| = 2n+3$, and $|E(G)| = 2n+2$.

We define $f : V(G) \to \{1,3,5,...,4n+5\}$ as follows

$f(u) = 1, f(u_i) = 2i+1, 1 \le i \le n, \ f(w) = 2n+3, f(v) = p$, where p is the largest prime number such that $2n+5 \le p \le 4n+5.$ The remaining vertices $v_i (1 \le i \le n)$ are assigned the labels $2n+5,...,4n+5$ in any order except p. We observe that from the above labeling pattern, we have, Therefore $|e_f(0) - e_f(1)| \le 1$.

Hence G is a vertex odd divisor cordial graph.

Example 3.4. Vertex odd divisor cordial labeling for $G = \prec B_{6,6} : w \succ$ is shown in Figure 2.

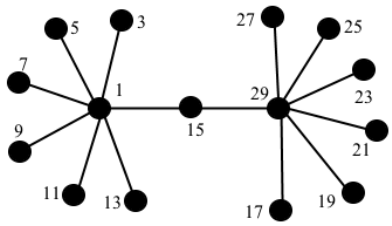

Figure 2: Vertex odd divisor cordial labeling for $G = \prec B_{6,6} : w \succ$

Theorem 3.5. $G = K_{1,n} * P_{n+2}$ is a vertex odd divisor cordial graph.

Proof: Let $G = K_{1,n} * P_{n+2}$. Let u be the apex vertex of $K_{1,n}$, and the remaining vertices of G are labeled as $u_1, u_2, u_3, ..., u_n$ respectively. Then $|V(G)| = 2n+3$, and $|E(G)| = 2n+2$ .We define $f : V(G) \to \{1,3,5,...,4n+5\}$ as follows $f(u) = 1, f(u_i) = 2i+1 (1 \le i \le 2n+1).$ In view of above-defined labeling pattern, we have, Therefore $|e_f(0) - e_f(1)| \le 1$.Hence G is a vertex odd divisor cordial graph.

Example 3.6. Vertex odd divisor cordial labeling for $G = K_{1,5} * P_7$ is shown in Figure 3 .

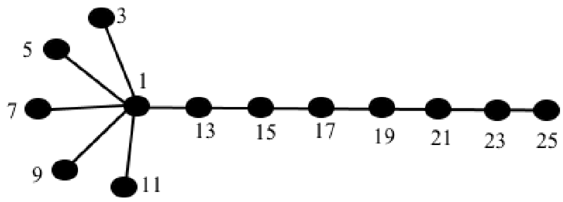

Figure 3: Vertex odd divisor cordial labeling for $G = K_{1,5} * P_7$

Theorem 3.7. $G = K_{2,n} \odot u_2(K_1)$ is a vertex odd divisor cordial graph.

Proof: Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n} \odot u_2(K_1)$ such that $V_1 = u_1, u_2$ and a pendant vertex w is attached with u_2. Then $|V(G)| = n+3$, and $|E(G)| = 2n+1$.
We define $f : V(G) \to \{1,3,5,...,2n+5\}$, as follows
$f(u) = 1$, $f(u_i) = 2i+1, 1 \le i \le n$. $f(u_2) = p$ where p is the largest prime number such that $p \le 2n+5$ and the remaining label is assigned to the vertex w. In view of above-defined labeling pattern, we have $e_f(0) = n+1$ and $e_f(1) = n$. Therefore $|e_f(0) - e_f(1)| \le 1$.Hence G is vertex odd divisor cordial labeling graph.

Example 3.8. Vertex odd divisor cordial labeling for $G = K_{2,7} \odot u_2(K_1)$ is shown in Figure 4.

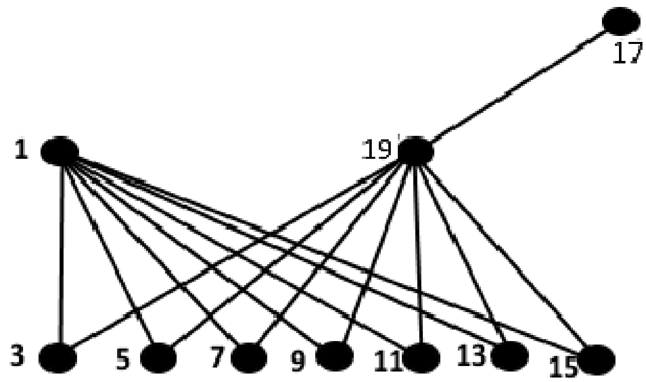

Figure 4: Vertex odd divisor cordial labeling for $G = K_{2,7} \odot u_2(K_1)$

Theorem 3.9. $G = \prec K_{1,n} \blacktriangle K_{1,n} \succ$ is vertex odd divisor cordial graph.

Proof: Let be the graph $\prec G_1 \blacktriangle G_2 \succ$. Let $u_1, u_2, u_3, ..., u_n$ be the vertices of G_1 and let $v_1, v_2, v_3, ..., v_n$ be the vertices of G_2. Let u, v be the apex vertices of G_1 and G_2. Let w be the new vertex joining the apex vertices of G_1 and G_2 in G.
Then $|V(G)| = 2n+3$, and $|E(G)| = 2n+3$.
We define $f : V(G) \to \{1,3,5,...,4n+5\}$ as follows
$f(u) = 1$, $f(u_i) = 2i+1 \quad (1 \le i \le n)$, $f(v) = p$ where p is the largest prime number such that $2n+3 \le p \le 4n+5$. The remaining vertices $v_i (1 \le i \le n)$ and w are assigned the labels $2n+3, ..., 4n+5$ in any order except p.
From the above labeling pattern, Thus $e_f(0) = n+1$ and $e_f(1) = n+2$.
Therefore $|e_f(0) - e_f(1)| \le 1$.
Hence G is a vertex odd divisor cordial graph.

Example 3.10. Vertex odd divisor cordial labeling for $G = \prec K_{1,8} \blacktriangle K_{1,8} \succ$ is shown in Figure 5.

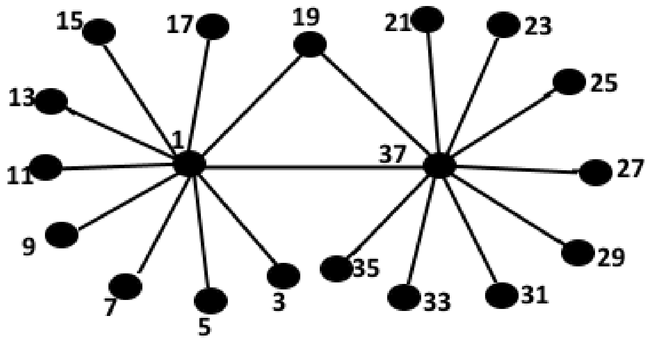

Figure 5: Vertex odd divisor cordial labeling for $G = \prec K_{1,8} \blacktriangle K_{1,8} \succ$

4. Conclusion

The vertex odd divisor cordial labeling is a variation of cordial labeling. It is very interesting to study graph or families of graph which are vertex odd divisor cordial as all the graphs do not admit vertex odd divisor cordial labeling. In this paper, we proved that the graphs $D_2(K_{1,n})$, $K_{2,n} \odot u_2(K_1)$, $\prec K_{1,n} \blacktriangle K_{1,n} \succ$, $\prec B_{n,n} : w \succ$ and $K_{1,n} * P_{n+2}$, are vertex odd divisor cordial graphs.

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