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Generalized Maximal Closed Sets in Topological Space

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Abstract. In this paper, we introduce and study generalized maximal closed sets in topological space and obtain some of their properties. A subset A of X is said to be generalized maximal closed (briefly g-m_a closed) set in a topological space (X, τ), if cl (A) $\subseteq$ U whenever A $\subseteq$U and U is maximal open set in X.

Keywords: Minimal closed, generalized minimal closed, maximal open set, ω-closed set

AMS Mathematics Subject Classification (2010): 54A05, 54B05

1. Introduction and preliminaries

The notion of closed set is fundamental in the study of topological spaces. In 1970, Levine [1] introduced the concept of generalized closed sets in topological spaces by comparing the closure of a subset with its open supersets. Further the study of g-closed sets was continued by Dunham and Levine [1]. Maximal open sets and Minimal open sets were studied and introduced by Nakaoka and Oda [3,4,5]. Benchalli, Banasode and Siddapur introduced and studied generalized minimal closed sets in topological spaces [2]. Further Banasode and Desurkar introduced and studied generalized minimal closed sets in bitopological spaces [7].

Throughout this paper (X, τ) represents a nonempty topological space on which no separation axioms are assumed unless otherwise explicitly stated.

For a subset A of a topological space (X,τ) cl (A), int (A) and A^c denote the closure of A, the interior of A and the complement of A in (X,τ) respectively. Let us recall the following definitions, which are useful in the sequel.

Here $int^*(A)$ denotes the interior of generalized open set A and $cl^*(A)$ denotes the closure of generalized closed set A.

Definition 1.1. [4] A proper nonempty subset A of a topological space (X,τ) is called
(i) a minimal open (resp. minimal closed) set if any open (resp. closed) subset of X which is contained in A, is either A or ϕ
(ii) a maximal open (resp. maximal closed) set if any open (resp. closed) set which contains A, is either A or X.

Definition 1.2. [1] A subset A of a topological space (X,τ) is called
(i) a generalized closed (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is an open set in X.
(ii) a generalized open (briefly g-open) set iff A^c is a g-closed set.
(iii) a ω-closed set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a semi open set in (X,τ).
(iv) an ω-open set iff A^c is a ω-closed set.
(iii) [2] a generalized minimal closed (briefly g-mi closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a minimal open set in X.

2. Generalized maximal closed sets

Definition 2.1. A subset A of a topological space (X,τ) is said to be generalized maximal closed (briefly g-m_a closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a maximal open set in X.

Theorem 2.2. Every g-m_a closed sets are ω- closed set.
Proof: Let V be a g-m_a closed set . By definition 2.1 cl(V) $\subseteq$ U whenever V $\subseteq$ U and U is maximal open set. We know that every maximal open set is open and also every open sets are semi-open sets. This implies U is a semi-open set. Therefore cl(V) $\subseteq$ U, whenever V $\subseteq$ U and U is semi-open set. Hence V is ω- closed set.

Remark 2.3. The converse of the above theorem is not true.

Example 2.4. Let X={a b c} with τ = {X, ϕ, {a}, {c}, {a c}, {a b}}
m_a - open sets = { a c}, {a b}
open sets= X, ϕ, {a}, {c}, {a c}, {a b}
closed sets = X, ϕ, {b}, {c}, {a b}, {b c}
g-m_a closed sets = ϕ, {b}, {c}, {a b}
ω- closed sets = ϕ, {b}, {c}, {a b}, {b c}, X
{b c} is ω- closed set but not g-m_a closed set.

Theorem 2.5. Every g-m_a closed set is g-closed set.
Proof: Let V be g-ma closed set. By Definition 2.1 cl(V) $\subseteq$ U. Whenever V $\subseteq$ U &U is maximal open set. We know that every maximal open set is open. This implies U is an open set. Therefore cl(V) $\subseteq$ U, whenever V $\subseteq$ U &U is an open set . Therefore V is g-closed set.

Remark 2.6. The converse of the above theorem is not true.

Example 2.7. Let X = {a b c} with τ = {X, ϕ,{a}}
Closed set = X, ϕ,{b c} ; Maximal open = {a}
g-m_a closed set = ϕ
g-closed set = ϕ,{b},{c},{a b},{b c},{a c},X.
Therefore {b} is a g-closed set but not g-m_a closed set.

Theorem 2.7. Every g-m_i closed set is g-m_a closed set.

Proof: Let V be a g-m_i closed set. By definition [2] cl(V) $\subseteq$ U. Whenever V $\subseteq$ U and U is minimal open set. Let A be a maximal open set then by [6] either U $\subseteq$ A or it is disconnected. Therefore U $\subseteq$ A. Thus cl(A) $\subseteq$ U $\subseteq$ A where A is a maximal open set. Thus A is g-m_a closed set.

Remark 2.8. The converse of the above theorem is not true.

Example 2.9. Let X={a,b,c,d} and τ= {X,ϕ,{a},{a,b},{c,d}{a,c,d}}
g-m_i closed sets : ϕ,{c},{d},{c,d}
g-m_a closed sets: ϕ,{b},{c},{d},{a,b},{c,d}
Here {b} and {a,b} are g-m_a closed sets but not g-m_i closed sets.

Remark 2.10. g-m_a closed, closed sets m_a-closed sets and α-closed sets are independent.

Example 2.11. Let X= {a b c d} with τ = {X, ϕ, {a},{a,b},{c,d},{a,c,d}}
Open sets = X, ϕ, {a}, {a,b}, {c,d}, {a,c,d}
Closed sets = X, ϕ, {b}, {a,b}, {c,d}, {b,c,d}
m_a – closed = {a,b},{b,c,d}
g-m_a closed = ϕ, {b}, {c}, {d}, {a,b}, {c,d}
g-m_i – open= X,{a,b}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}.
α-closed sets = ϕ,{b},{a,b},{c,d},{b,c,d}

Remark 2.12. From the above implications we have the following as given below

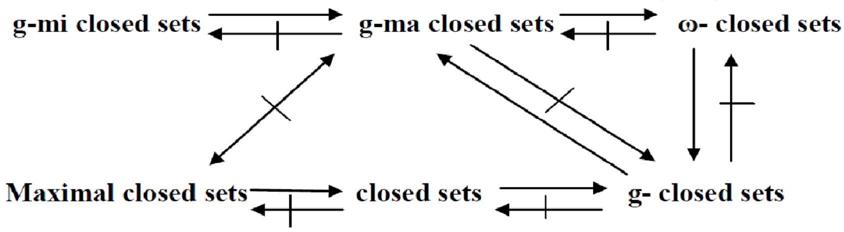

Figure 1:

Theorem 2.13. Intersection of any two g-m_a closed is g-m_a closed

Proof: Let A & B be any two non-empty g-m_a closed set. Then by definition 2.1 cl(A) $\subseteq$ U, whenever A $\subseteq$ U where U is m_a –open, also cl(B) $\subseteq$ U, whenever B $\subseteq$ U where U is m_a–open. We know that cl(A $\cap$ B) $\subseteq$ cl(A) $\cap$ cl(B) also, since cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is m_a –open& cl(B) $\subseteq$ U, whenever B $\subseteq$ U, U is m_a –open.Thus cl(A $\cap$ B) $\subseteq$ U whenever A $\subseteq$ U & B $\subseteq$ U, U is m_a –open. Therefore cl(A $\cap$ B) $\subseteq$ U whenever A $\cap$ B $\subseteq$ U &U is m_a –open. Therefore A $\cap$ B is g-m_a closed sets.

Remark 2.14. Union of any two g-m_a closed sets may not be g-m_a closed sets.

Example 2.15. From Example 2.4,clearly {b} & {c} are g-m_a closed sets but {b c} is not g-m_a closed sets.

Theorem 2.16. If A and B are any two g-m_a closed then cl(A $\cup$ B) = cl(A) $\cup$ cl(B)
Proof: Since, A $\subseteq$ A$\cup$ B we have cl(A) $\subseteq$ cl(A$\cup$ B) and since B $\subseteq$ A $\cup$ B we have cl(B) $\subseteq$ cl(A$\cup$ B). Therefore cl(A) $\cup$ cl(B) $\subseteq$ cl(A$\cup$ B) since cl(A) and cl(B) are g-m_a closed sets. Therefore A $\subseteq$ cl(A) and B $\subseteq$ cl(B) this implies A $\cup$ B $\subseteq$ cl(A) $\cup$ cl(B).Thus cl(A) $\cup$ cl(B) is the closed containing A $\cup$ B. Since cl(A$\cup$ B) is the smallest closed set containing A $\cup$ B. Therefore cl(A$\cup$ B) $\subseteq$ cl(A) $\cup$ cl(B) Hence cl(A$\cup$ B)=cl(A) $\cup$ cl(B)

Theorem 2.17. If A is g-m_a closed in a top space (X,τ) then cl(A)- A contains no non empty minimal closed set.
Proof: Let F be any minimal closed subset of cl(A)-A. Then F^c is a maximal open subset. Therefore F $\subseteq$ cl(A)-A=cl(A) $\cap$ A^c
This implies F $\subseteq$ cl(A) and F $\subseteq$ A^c, since cl(A) $\cap$ A^c $\subseteq$ cl(A) and cl(A) $\cap$ A^c $\subseteq$ A^c. Therefore F $\subseteq$ A^c this implies A $\subseteq$ F^c, where F^c is maximal open set. Since A is g-m_a closed set, we have by the definition cl(A) $\subseteq$ F^c whenever A $\subset$ F^c & F^c is maximal open. Since cl(A) $\subseteq$ F^c this implies F $\subseteq$ $[cl(A)]^c$ .Therefore F= ϕ

Theorem 2.18. If A is a g-m_a closed & A $\subseteq$ B $\subseteq$ cl(A) then B is a g-m_a closed set in a topological space in (X,τ).
Proof: Let B be any set such that B $\subseteq$ U&U is maximal open set.From the hypothesis A $\subseteq$ B $\subseteq$ cl(A).Since A is g-m_a closed, then by the definition 2.1we have cl(A) $\subseteq$ U whenever A $\subseteq$ U & U is maximal open set.Since A $\subseteq$ B $\subseteq$ cl(A) & cl(A) $\subseteq$ U this implies B $\subseteq$ cl(A) cl(B) $\subseteq$ cl(cl(A))=cl(A) this implies cl(B) $\subseteq$ cl(A) $\subseteq$ U. Therefore cl(B) $\subseteq$ U, whenever B $\subseteq$ U & U is maximal open set. Therefore is g-m_a closed set.

Theorem 2.19. If A is a g-m_a closed set in a topological space (X,τ) then cl(A)-A has no non empty minimal closed set.
Proof : Let U be minimal closed subset of cl(A)-A , then U^c is maximal open set. U $\subseteq$ cl(A)-A this implies U $\subseteq$ cl(A) $\cap A^c$. Thus U $\subseteq$ cl(A) and U $\subseteq$ A^c this implies A $\subseteq$ U^c where U^c is maximal open set. Since A is g-m_a closed set then by definition 2.1 cl(A) $\subseteq$ U^c which implies U $\subseteq$ $[cl(A)]^c$ also U $\subseteq$ cl(A) .Therefore U $\subseteq$ $[cl(A)]^c$ $\cap$ cl(A) = ϕ. Therefore U = ϕ.

Theorem 2.20. If A is a g-m_a closed set in a topological space (X,τ) then cl(A)-A has no non emptyclosed subset.
Proof: The proof is omitted has it is obvious from the above Theorem 2.19.

Remark 2.21. If A is the only maximal open set in a topological space then g-m_a closed set is a null set.
Example 2.22. Let X={a,bc,d} and τ={ϕ,{a},{b},{a,b},{a,b,c}}.Maximal open set={a,b,c} therefore g-m_a closed set={ ϕ }.

Remark 2.23. If A is maximal open set and g-m_a closed set then A is a closed set.

Example 2.24. From example 2.11 {a,b} is both g-ma closed set and maximal open set. Thus {a,b} is a closed set in X.

3. Generalized minimal open sets

Definition 3.1. A subset U of X is said to be generalized minimal open sets iff its complement is generalized maximal closed set.

Remark 3.2. For any subset A of X,$int^*(cl^*(A)-A) = \phi$

Remark 3.3. For any subset A of X , $cl^*(X-A) = X-int^*(A)$

Theorem 3.4. Every g-$m_i$open set is g-open set.
Proof: This follows from the definition 3.1 and theorem 2.5.

Remark 3.5. The converse of the above theorem is not true.

Example 3.6. Let X={a,b,c} and τ={X,ϕ,{a},{b,c}}.Then the set A={b} is g-open set but not g-m_i open set.

Theorem 3.7. Every g-m_i open set is ω- open set.
Proof: This follows from the definition 3.1 and Theorem 2.2.

Remark 3.8. The converse of the above Theorem is not true.

Example 3.9. From Example 3.6 A={b} is ω- open set but not g-m_i open set.

Remark 3.10. From the above implications we have the following as given below

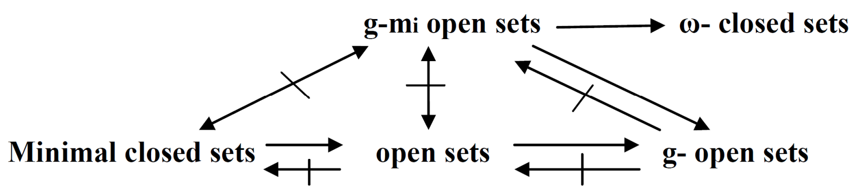

Figure 2:

Theorem 3.11. If B is g-m_i open iff $F \subseteq int(B)$ whenever $F \subseteq B$ and F is minimal closed set.

Proof: Let B be a g-m_i open set and F be a minimal closed set such that $F \subseteq B$ which implies $X- B \subseteq X- F$, where X- F is maximal open set. Let $X- B \subseteq cl^*(X-B) \subseteq X- F$ this implies $X-int^*(B) \subseteq X- F$. Thus $F \subseteq int^*(B) \subseteq int(B)$. Therefore $F \subseteq int (A)$.

Suppose F is minimal closed and $F \subseteq int B$ whenever $F \subseteq B$. Let $X- B \subseteq U$ where U is maximal open sets. Then $X- U \subseteq B$ where X- U is minimal closed set. Therefore by the hypothesis $X- U \subseteq int(B)$ which implies $X- int(B) \subseteq U$ which implies $cl(X- B) \subseteq U$ Therefore X- B is generalized maximal closed set . Hence B is generalized minimal open set.

Theorem 3.12. If int A $\subseteq$ B $\subseteq$ A and A is g-m_i open then B is g-m_i open.
Proof: Let int A $\subseteq$ B $\subseteq$ A and A is g-m_i open set. Then $A^c \subseteq B^c \subseteq (int(A))^c$ this implies $A^c \subseteq B^c \subseteq cl(A)^c$.Since A^c is g-m_a closed then by Theorem2.18 ,B^c isg-m_a closed.Thus B is g-m_i open set.

Theorem 3.13. If A isg-m_i open set in X then U=X whenever U is an open set and int(A) $\cup$ $A^c \subseteq$ U.
Proof: Let A be g-m_i open set in X. Let U be an open set and int(A) $\cup$ $A^c \subseteq$ U which implies $U^c \subseteq (int\ A)^c \cap$ A. Since A^c is g-m_a closed set and U^c is closed set ,it follows from Theorem 2.20 that $U^c = \phi$. Therefore U = X.

Remark 3.14. If A is g-m_a closed set then $cl^*(A)$ - A is g-m_i open set.

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