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On Mann-type Implicit Iteration Method for a Family of α-demicontractive Mappings in Hilbert Spaces

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Abstract. The purpose of this paper is to establish the strong convergence theorems for α-demicontractive mappings using Mann-type implicit iteration process under suitable conditions in Hilpbert spaces.

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1. Introduction

Let H be a Hilbert space and K is a closed convex subset of H. A mapping $T : K \to K$ is a (possibly nonlinear) mapping. The set of fixed points of T is denoted by $F(T)$. The mapping T is said to

- Nonexpansive if, $\|Tx - Ty\| \leq \|x - y\|$, for all $x, y \in K$;
- Pseudocontractive if,
  $\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|(I - T)x - (I - T)y\|^2$, for all $x, y \in K$;
- Strongly pseudocontractive if there exists $k \in (0, 1)$ such that
  $\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2$, for all $x, y \in K$;
- Demicontractive if $F(T) \neq \phi$ and
  $\|Tx - p\|^2 \leq \|x - p\|^2 + k\|x - Tx\|^2$, for all $x \in K$ and $p \in F(T)$;
- Hemicontractive if $F(T) \neq \phi$ and
  $\|Tx - p\|^2 \leq \|x - p\|^2 + \|x - Tx\|^2$, for all $x \in K$ and $p \in F(T)$.

It follows from the definition that a pseudocontractive or a demicontractive mapping is hemicontractive. Many research paper has been published on the iterative approximation of fixed points of Lipschitz strongly pseudocontractive mappings using the Mann iteration process in Hilbert spaces and further extended to more general Banach spaces [1, 3, 4, 6, 5, 7, 9, 10, 11, 20, 23].

Recall that the Mann [17] iteration formula is given by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n,\ x_0 \in K,$$

where $\{\alpha_n\}$ is a real sequence in [0,1] satisfying some appropriate conditions.

In 1974, Ishikawa [14] introduced the following iteration process:

$$\begin{cases} x_1 \in K, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \quad n \geq 1 \end{cases}$$

where the two sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy some appropriate conditions and proved strong convergence theorem for Lipschitzian pseudocontractive mapping in Hilbert space and Qihou [18] extended this result to more general class of Lipschitz hemicontactive mapping.

The following iteration is due to Liu [15]. The sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in K, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n + v_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n + u_n\,, \quad n \geq 1 \end{cases}$$

for each $n \geq 1$ where $\{\alpha_n\}$, $\{\beta_n\} \in [0,1]$ satisfying appropriate conditions and $\sum\|u_n\| < \infty$, $\sum\|v_n\| < \infty$, known as Ishikawa iteration process with errors. The sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in K, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n + u_n, \end{cases}$$

for each $n \geq 1$ where $\{\alpha_n\} \in [0,1]$ satisfying appropriate conditions and $\sum\|u_n\| < \infty$, known as Mann iteration process with errors.

In 1998, Xu [25] introduced the following iteration process:

$$\begin{cases} x_1 \in K, \\ y_n = a_n{}' x_n + b_n{}' T x_n + c_n{}' v_n, \\ x_{n+1} = a_n x_n + b_n T y_n + c_n u_n\,, \quad n \geq 1 \end{cases}$$

for each $n \geq 1$ where $\{u_n\}, \{v_n\}$ are the bounded sequences in K and $\{a_n\}$, $\{b_n\}$, $\{c_n\}, \{a_n{}'\}, \{b_n{}'\}$ and $\{c_n{}'\}$ are the sequences in [0,1] such that $a_n + b_n + c_n = a_n{}' + b_n{}' + c_n{}' = 1$ for each $n \geq 1$ is known as Ishikawa iteration with errors in the sense of Xu.

The following Theorem is proved by Chidume and Moore [8].

Theorem 1.1. [8] Let K be a compact convex subset of a real Hilbert space H and $T : K \to K$ be a continuous hemicontractive mapping. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}, \{a_n{}'\}, \{b_n{}'\}$ and $\{c_n{}'\}$ be the real sequences in [0,1] satisfying the following conditions:

(i) $a_n + b_n + c_n = a_n{}' + b_n{}' + c_n{}' = 1$;

(ii) $lim b_n = lim b_n{}' = 0$;

(iii) $\sum c_n < \infty$, $\sum c_n{}' < \infty$;

(iv) $\sum \alpha_n\beta_n < \infty$, and $\sum \alpha_n\beta_n\delta_n < \infty$, where $\delta_n = \|x_n - Ty_n\|^2$;

(v) $0 \leq \alpha_n \leq \beta_n < 1$ for each $n \geq 1$, where $\alpha_n := b_n + c_n$ and $\beta_n := b_n{}' + c_n{}'$

For arbitrary $x_1 \in K$, the sequence $\{x_n\}$ defined by

$$\begin{cases} x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \\ y_n = a_n' x_n + b_n' T x_n + c_n' v_n \end{cases}$$

for each $n \geq 1$ where $\{u_n\}, \{v_n\}$ are the arbitrary sequences in K. Then $\{x_n\}$ converges strongly to a fixed point of T.

Remark 1.2.

(i) Borwein and Borwein [2] identified an example of a Lipschitz map (which is not necessarily pseudocontractive) with a unique fixed point for which the Mann iteration fails to converge.

(ii) Hicks and Kubicek [12] identified an example of a discontinuous pseudocontraction with a unique fixed point for which the Mann iteration does not always converge.

(iii) Chidume and Mutangadura [9] identified an example of continuous Lipschitz pseudocontraction with a unique fixed point for which every non trivial Mann iteration fails to converge.

In 2007, Rafiq [19] proved the following result.

Theorem 1.3. [19] Let K be a compact convex subset of a real Hilbert space H and $T : K \to K$ be a hemicontractive mapping. Let $\{\alpha_n\}$ be a real sequence in [0,1] satisfying $\{\alpha_n\} \subset [\delta, 1-\delta]$ for some $\delta \in (0,1)$. For arbitrary $x_0 \in K$, the sequence $\{x_n\}$ is defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T x_n. \tag{1}$$

Then $\{x_n\}$ converges strongly to a fixed point of T.

But, Song [21] observed that there is a gap in the iteration process (1) for hemicontractive mapping T and proved the following theorem.

Theorem 1.4. [21] Suppose K is a compact convex subset of a real Banach space E and $T : K \to K$ is a continuous pseudocontractive mapping such that $F(T) \neq \phi$. Assume that $\{\alpha_n\} \subset (0,1)$ is a real sequence satisfying the condition $lim_{n\to\infty}\alpha_n = 0$. Let $x_0 \in K$ and let $\{x_n\}$ be defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T x_n, \quad n \geq 0.$$

Then $\{x_n\}$ strongly converges to a fixed point of T.

In 2013, Hussain et al. [13] introduced the following Mann-type implicit iteration associated with a family of continuous hemicontractive mappings to prove a strong convergence result in Hilbert spaces.

$$\begin{cases} x_0 \in K, \\ x_n = \alpha_n x_{n-1} + \sum_{i=1}^{m} \beta_n{}^i T_i x_n \end{cases} \tag{2}$$

for each $n \geq 1$ where $\alpha_n,\ \beta_n{}^i \in [0,1], i = 1,2,\dots,m$, are such that $\alpha_n + \sum_{i=1}^{m} \beta_n{}^i = 1$ and some appropriate conditions hold.

Theorem 1.5. [13] Let K be a compact convex subset of a real Hilbert space H and $T_i: K \to K,\ i = 1,2,\dots,m$, be a family of continuous hemicontractive mappings. Let $\alpha_n,\ {\beta_n}^i \in [0,1]$ be such that $\alpha_n + \sum_{i=1}^{m} {\beta_n}^i = 1$ and satisfying $\{\alpha_n\},\ {\beta_n}^i \subset [\varepsilon, 1-\varepsilon]$ for some $\varepsilon \in (0,1), i = 1,2,\dots,m$. Then, for arbitrary $x_0 \in K$, the sequence $\{x_n\}$ defined by (2) converges strongly to a common fixed point in $\bigcap_{i=1}^{m} F(T_i) \neq \phi$.

In 2011, Maruster and Maruster [16] introduced the concept of α-demicontractivity in Hilbert spaces and obtained some strong convergence theorems.

Definition 1.6. [16] Let K be a closed convex subset of Hilbert space H, then a mapping from $T:\ K \to K$ is said to be α-demicontractive if for some $\alpha \geq 1$,

$$\|Tx - \alpha p\|^2 \leq \|x - \alpha p\|^2 + k\|x - Tx\|^2,\ k \in (0,1)$$

for all $x \in K$ and $p \in F(T)$.

Remark 1.7. [16] If T is α-demicontractive then αp is a fixed point of T.

Motivated by the above definition and remark, the purpose of this paper is to establish strong convergence results for family of continuous α-demicontractive mappings using the Mann-type implicit iteration process (2) given by Hussain et al. [13] which extend the corresponding results of Hussain et al. [13].

2. Main results

In the sequel, we need the following Lemmas.

Lemma 2.1. [22] Suppose that $\{\rho_n\}, \{\sigma_n\}$ are two sequences of nonnegative numbers such that, for some real number $N_0 \geq 1$, $\rho_{n+1} \leq \rho_n + \sigma_n$ for all $n \geq N_0$. Then we have the following:

(i) If $\sum \sigma_n < \infty$, then $lim\ \rho_n$ exists.

(ii) If $\sum \sigma_n < \infty$ and $\{\rho_n\}$ has a subsequence converging to zero, then $lim\ \rho_n = 0$.

Lemma 2.2. [24] For all $x, y \in H$ and $\lambda \in [0,1]$, the following well known identity holds: $\|(1-\lambda)x + \lambda y\|^2 = (1-\lambda)\|x\|^2 + \lambda\|y\|^2 - \lambda(1-\lambda)\|x - y\|^2$.

Lemma 2.3. [13] Let H be a Hilbert space. Then, for all $x,\ x_i \in H, i = 1,2,\dots m$,

$$\left\|\gamma x + \sum_{i=1}^{m} \delta^i x_i\right\|^2 = \gamma\|x\|^2 + \sum_{i=1}^{m} \delta^i\|x_i\|^2 - \sum_{i=1}^{m} \gamma\delta^i\|x_i - x\|^2 - \sum_{\substack{i,j=1 \\ i\neq j}}^{m} \delta^i\delta^j\left\|x_i - x_j\right\|^2$$

where $\gamma, \delta^i \in [0,1], i = 1,2,\dots,m$ and $\gamma + \sum_{i=1}^{m} \delta^i = 1$.

Theorem 2.4. Let K be a compact convex subset of a real Hilbert space H and $T_i: K \to K, i = 1,2,\dots,m$, be a family of continuous α-demicontractive mappings. Let $\gamma_n, {\delta_n}^i \in [0,1]$ be such that $\gamma_n + \sum_{i=1}^{m} {\delta_n}^i = 1$ and satisfying $\{\gamma_n\},\ {\delta_n}^i \subset [\varepsilon, 1-\varepsilon]$ for some

$\varepsilon \in (0,1), i = 1,2,\dots,m$. Then, for arbitrary $x_0 \in K$, the sequence $\{x_n\}$ defined by

$$\begin{cases} x_0 \in K, \\ x_n = \gamma_n x_{n-1} + \sum_{i=1}^{m} {\delta_n}^i T_i x_n \end{cases} \tag{3}$$

converges strongly to a common fixed point in $\bigcap_{i=1}^{m} F(T_i) \neq \phi$.

Proof: Let $\alpha p \in \bigcap_{i=1}^{m} F(T_i)$. Since each $T_i, i = 1,2,\dots,m$ are α-demicontractive, therefore for some $\alpha > 1$, we obtain

$$\|T_i x_n - \alpha p\|^2 \leq \|x_n - \alpha p\|^2 + k\|x_n - T_i x_n\|^2 \tag{4}$$

From (3), Lemma 2.3 and (4), we have

$$\|x_n - \alpha p\|^2 = \left\|\gamma_n x_{n-1} + \sum_{i=1}^{m} {\delta_n}^i T_i x_n - \alpha p\right\|^2$$

$$= \left\|\gamma_n (x_{n-1} - \alpha p) + \sum_{i=1}^{m} {\delta_n}^i (T_i x_n - \alpha p)\right\|^2$$

$$= \gamma_n \|x_{n-1} - \alpha p\|^2 + \sum_{i=1}^{m} {\delta_n}^i \|T_i x_n - \alpha p\|^2 - \sum_{i=1}^{m} \gamma_n {\delta_n}^i \|x_{n-1} - T_i x_n\|^2 - \sum_{\substack{i,j=1 \\ i \neq j}}^{m} {\delta_n}^i {\delta_n}^j \|T_i x_n - T_j x_n\|^2$$

$$\leq \gamma_n \|x_{n-1} - \alpha p\|^2 + \sum_{i=1}^{m} {\delta_n}^i \|T_i x_n - \alpha p\|^2 - \sum_{i=1}^{m} \gamma_n {\delta_n}^i \|x_{n-1} - T_i x_n\|^2 \tag{5}$$

From (4) and (5), we have

$$\|x_n - \alpha p\|^2 \leq \gamma_n \|x_{n-1} - \alpha p\|^2 + \sum_{i=1}^{m} {\delta_n}^i \|x_n - \alpha p\|^2 + k \sum_{i=1}^{m} {\delta_n}^i \|x_n - T_i x_n\|^2 - \sum_{i=1}^{m} \gamma_n {\delta_n}^i \|x_{n-1} - T_i x_n\|^2 \tag{6}$$

Also, we have

$$\|x_n - T_i x_n\|^2 = \left\|\gamma x_{n-1} + \sum_{i=1}^{m} {\delta_n}^i T_i x_n - T_i x_n\right\|^2 = {\gamma_n}^2 \|x_{n-1} - T_i x_n\|^2 \tag{7}$$

From (6) and (7), we have

$$\|x_n - \alpha p\|^2 \leq \gamma_n \|x_{n-1} - \alpha p\|^2 + \sum_{i=1}^{m} {\delta_n}^i \|x_n - \alpha p\|^2 - \sum_{i=1}^{m} \gamma_n {\delta_n}^i (1 - k\gamma_n) \|x_{n-1} - T_i x_n\|^2$$

$$\leq \|x_{n-1} - \alpha p\|^2 - \sum_{i=1}^{m} \gamma_n {\delta_n}^i (1 - k\gamma_n) \|x_{n-1} - T_i x_n\|^2$$

From condition, $\{\gamma_n\}$, ${\delta_n}^i \subset [\varepsilon, 1 - \varepsilon]$ for some $\varepsilon \in (0,1), i = 1,2,\dots,m$, we obtain

$$\|x_n - \alpha p\|^2 \leq \|x_{n-1} - \alpha p\|^2 - \varepsilon(1 - k\varepsilon) \sum_{i=1}^{m} \|x_{n-1} - T_i x_n\|^2 \tag{8}$$

for all fixed points $\alpha p \in \bigcap_{i=1}^{m} F(T_i)$.

From (8), we have

$$\varepsilon(1-k\varepsilon)\sum_{i=1}^{m}\|x_{n-1}-T_i x_n\|^2 \leq \|x_{n-1}-\alpha p\|^2 - \|x_n-\alpha p\|^2$$

$$\varepsilon(1-k\varepsilon)\sum_{j=1}^{\infty}\left\|x_{j-1}-T_i x_j\right\|^2 \leq \sum_{j=1}^{\infty}\left(\left\|x_{j-1}-\alpha p\right\|^2 - \left\|x_j-\alpha p\right\|^2\right),$$

for all $i = 1,2,\ldots,m$

$$= \|x_0-\alpha p\|^2$$

$$\textstyle\sum_{j=1}^{\infty}\left\|x_{j-1}-T_i x_j\right\|^2 < \infty, \text{ for all } i = 1,2,\ldots,m \tag{9}$$

This implies, $lim_{n\to\infty}\|x_{n-1}-T_i x_n\| = 0$, for all $i = 1,2,\ldots,m$

From (7), $lim_{n\to\infty}\|x_n-T_i x_n\| = 0$, for all $i = 1,2,\ldots,m$

Since K is compact, there is a subsequence $\left\{x_{n_j}\right\}$ of $\{x_n\}$ which converges to a common fixed point of $\bigcap_{i=1}^{m}F(T_i)$, say αp. Since (8) holds for all fixed points of $\bigcap_{i=1}^{m}F(T_i)$, we have

$$\|x_n-\alpha p\|^2 \leq \|x_{n-1}-\alpha p\|^2 - \varepsilon(1-k\varepsilon)\sum_{i=1}^{m}\delta_n{}^i\|x_{n-1}-T_i x_n\|^2$$

From (9) and Lemma 2.1,

$$\|x_n-\alpha p\| \to 0 \text{ as } n\to\infty.$$

This completes the proof.

Theorem 2.5. Let $H, K, T_i, i = 1,2,\ldots,m,$ be as in Theorem 2.4 and $\{\gamma_n\}$, $\delta_n{}^i \in [0,1]$ be such that $\gamma_n + \sum_{i=1}^{m}\ \delta_n{}^i = 1$ and satisfying $\{\gamma_n\}$, $\delta_n{}^i \subset [\varepsilon, 1-\varepsilon]$ for some $\varepsilon \in (0,1), i = 1,2,\ldots,m$. If $P_K: H\to K$ is the projection operator of H onto K, then the sequence $\{x_n\}$ defined iteratively by $x_n = P_K\left(\gamma_n x_{n-1} + \sum_{i=1}^{m}\ \delta_n{}^i T_i\, x_n\right)$ for each $n \geq 0$ converges strongly to a common fixed point in $\bigcap_{i=1}^{m}F(T_i) \neq \phi$.

Proof: Since the mapping P_K is nonexpansive (see [1]) and K is a Chebyshev subset of H, therefore P_K is a single valued mapping. We have,

$$\|x_n-\alpha p\|^2 = \left\|P_K\left(\gamma_n x_{n-1} + \sum_{i=1}^{m}\delta_n{}^i T_i\, x_n\right) - P_K\alpha p\right\|^2$$

$$\leq \left\|\gamma_n x_{n-1} + \sum_{i=1}^{m}\delta_n{}^i T_i\, x_n - \alpha p\right\|^2$$

$$\leq \left\|\gamma_n(x_{n-1}-\alpha p) + \sum_{i=1}^{m}\delta_n{}^i(T_i\, x_n - \alpha p)\right\|^2$$

$$\leq \gamma_n\|x_{n-1}-\alpha p\|^2 + \sum_{i=1}^{m}\delta_n{}^i\|x_n-\alpha p\|^2 - \sum_{i=1}^{m}\gamma_n(1-k\gamma_n)\delta_n{}^i\|x_{n-1}-T_i x_n\|^2$$

$$\|x_n-\alpha p\|^2 \leq \|x_{n-1}-\alpha p\|^2 - \sum_{i=1}^{m}\gamma_n(1-k\gamma_n)\delta_n{}^i\|x_{n-1}-T_i x_n\|^2$$

It follows from the fact that the set $K \cup T(K)$ is compact, the sequence $\{\|x_n - T_i x_n\|\}$ is bounded. Following the same argument as exactly the proof of Theorem 2.4, $\{x_n\}$

converges strongly to a common fixed point in $\bigcap_{i=1}^{m} F(T_i) \neq \phi$. This completes the proof.

Remark 2.6 For $m = 2$, we can choose the following control parameters:
$\gamma_n = \frac{1}{4} - \frac{1}{(n+2)^2}$, $\delta_n{}^1 = \frac{1}{4}$ and $\delta_n{}^2 = \frac{1}{2} + \frac{1}{(n+2)^2}$.

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