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A Bi-Criteria Multi-Index Bulk Transportation Problem

***Sungeeta Singh*[1], *Sudhir Kumar Chauhan*[2] and *Kuldeep*[3]**

[1]Department of Mathematics, Amity University, Gurugram
Haryana, India. E-mail: sungeeta2003@rediffmail.com
[2]Department of Mathematics, Amity School of Engineering and Technology
Bijwasan, New Delhi, India. E-mail:skchauhan@amity.edu
[3]Department of Mathematics, Amity University
Gurugram, Haryana, India. E-mail: k.tanwarmath@gmail.com
[3]Corresponding Author

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***Abstract*.** In the present work, bi-criteria multi-index bulk transportation problem (BCMIBTP) is considered. Multi-index bulk transportation problem (MIBTP) is an extension of bulk transportation problem (BTP). MIBTP exists when more than one type of products are produced at sources or when the products are supplied through various modes of transportation and the need of a destination must be fulfilled from only single source. In the present paper, cost-time trade-off relations are studied in MIBTP. A solution procedure is proposed to get the cost-time trade-off pairs in MIBTP. Initially, the proposed solution procedure determines the least cost of MIBTP and corresponding time and then the next cost-time trade-off pairs are determined. To illustrate the proposed solution procedure, an example is worked out.

Keywords: Bi-Criteria, Multi-index, Bulk Transportation, Transportation Problem, cost-time trade-off.

AMS Mathematics Subject Classification (2010): 90C08

1. Introduction

The transportation problem is a special class of linear programming problem related tothe distribution of number of units of a product from some sources to some destinations**.** Hitchcock [1] was first who developed the concept of transportation problem with the objective of minimizing thecost of transportation. Single objective transportation problems are discussed by several authors, namely, Balas [2], Bhatia et al. [3], Garfinkel and Rao [4], Hammer [5], Seshan and Tikekar [6] and Sharma and Swarup [7], Hakim [8], Ahmed et al. [9] and Pramila and Uthra [10] using different approaches.

Bulk transportation problem(BTP) is another version of transportation problem in which a source can supply the number of units of a homogeneous product to any number of destinations but the demand of a destination must be fulfilled from a single source. Maio and Roveda [11] were the first who introduced BTP in literature with the objective of minimizing the bulk cost of transportation. After that, Srinivasan and Thompson [12] studied the BTP with help of branch and bound method. Later on, Bhatia [13] discussed a note on time minimizing BTP.

Multi-Index Transportation Problem (MITP) is an extension of transportation problem having more than two indices. Such type of problems exists when more than one type of products are produced at sources or when the products are supplied through various modes of transportation. Schell [14] and Galler and Dwyer [15] were the first who introduced MITP in literature. After that authors, Haley [16], Junginer [17], Pandian and Anuradha [18] and Rautman et al. [19] discussed MITP through different approaches. Purusotham and Murthy [20] presented Lexi-Search algorithm to minimize thecost of MIBTP.

The transportation problems having both cost and time as objectives are known as bi-criteria transportation problems. Bhatia [21], Glickman and Berger [22], Aneja and Nair [23] and Prakash et al. [24] have studied bi-criteria transportation problem. Bi-Criteria BTP has been studied by Prakash and Ram [25], Prakash et al. [26]. BCMIBTP is not studied so far in the literature.

In this paper, an efficient solution procedure is proposed to study the cost-time trade-off relations in MIBTP. In Section 2, formulation of the problem is presented. Section 3, presents the solution procedure and in Section 4, steps of the proposed solution procedure are illustrated through a numerical example.

2. Formulation of the problem

Let there are m sources and n destinations and l facilities. The formulation of the BCMIBTP is as follows:

$$\text{Min}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\, y_{ijk} : y_{ijk} = 1,\ \max\{t_{ijk} : y_{ijk} = 1\}\right) \quad (1)$$

Subject to the constraints

$$\sum_{j=1}^{n}\sum_{k=1}^{l} b_j y_{ijk} \le a_i \quad (2)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} y_{ijk} = 1 \quad (3)$$

$$y_{ijk} = 1 \ or\ 0 \quad (4)$$

Here c_{ijk} is the bulk cost of transportation from ith source to jth destination availing the facility ‘k’, y_{ijk} is the decision variable assuming the value 1 or 0 depending upon whether the requirement of the jth destination is met or not met from the source ‘i’ availing the facility ‘k’, t_{ijk} is the bulk time of transportation from ith source to jth destination availing the facility ‘k’, a_i is the number of units of the product available at the ith source and b_j is the number of units of the product required at the jth destination.

Definition 2.1. Let C_1 be the minimum cost of BCMIBTP and T_1 be the associated time. Let Y_1 be the solution yielding this cost-time pair $[C_1, T_1]$. Let $C_2(> C_1)$ be another cost of BCMIBTP and $T_2(< T_1)$ be the minimum time of BCMIBTP associated with cost C_2. Then, the solution Y_2 providing the cost-time pair $[C_2, T_2]$ is said to be the subsequent efficient solution if there exist no other solution Y providing a cost-time pair $[C, T]$ s.t. $C_1 < C < C_2$ and $T_2 < T < T_1$. The pair $[C_2, T_2]$ provided by the solution Y_2 is the next efficient cost-time pair. In the same way, the subsequent efficient cost-time pairs may be determined.

3. Solution procedure

Steps of solution procedure:

Step 1. Cross-off the cells (i, j) from the table for which availability a_i is less than requirement b_j.

Step 2. Determination of minimum cost of bulk transportation for each mode of transportation:
Determine the minimum cost of bulk transportation problem P_1, C_{11}(say) and let T_{11} is the associated time of bulk transportation and Y_{11} be the solution providing the minimum cost C_{11} and corresponding time T_{11} for bulk transportation through facility k=1. Let Y_{12} be the solution providing the minimum cost C_{12} and corresponding time T_{12} for bulk transportation problem P_2.

Step 3. Select a minimum of the corresponding costs associated with decision variables for each destination through facility k = 1 and k = 2. Let Y_1 be the solution so obtained and let the associated cost and time of MIBTP are C_1 and T_1 respectively. Then (C_1, T_1) is the first efficient cost-time trade-off pair of the BCMIBTP

Step 4. To obtain next efficient cost-time trade-off pairs, remove those cells from the tables of problems P_1 and P_2 which have time more than or equal to T_1. Repeat steps 2 to 3 for the reduced problem, we obtain 2nd cost-time trade-off pair (C_2, T_2).

Step 5. Repeat the above steps until the reduced problem becomes infeasible.

4. Numerical problem

Consider a BCMIBTP which consist of 3 sources and 5 destinations. The requirements of destinations are fulfilled through two modes of transportation. The availabilities and requirements of the sources and destinations are 7, 8, 9 and 3, 5, 4, 6 and 2 respectively. Here the main problem comprises two subproblems P_1 and P_2. Bulk cost and bulk time of transportation from each source to each destination are given through two modes of transportation respectively. The first and second entry in each cell denotes bulk cost and bulk time of transportation respectively. The proposed method is applied to the problem to obtain all efficient cost-time trade-off pairs. The tableau representation of the BCMIBTP is given below.

Table 4.1: P_1(Representation of costs and times of BCMIBTP through 1st mode of transportation)

		Destinations					
		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	(10, 6)	(9, 7)	(12, 4)	(7, 3)	(8, 4)	7
	S_2	(11, 3)	(10, 4)	(14, 6)	(14, 7)	(12, 5)	8
	S_3	(8, 5)	(6, 6)	(10, 9)	(10, 4)	(13, 9)	9
	$b_j \rightarrow$	3	5	4	6	2	

Table 4.2: P_2 (Representation of costs and times of BCMIBTP through 2nd mode of transportation)

Destinations

		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	(12, 5)	(10, 6)	(14, 5)	(8, 4)	(7, 5)	7
	S_2	(9, 4)	(12, 5)	(13, 4)	(16, 6)	(13, 7)	8
	S_3	(7, 6)	(5, 7)	(10, 8)	(12, 5)	(11, 6)	9
	$b_j \rightarrow$	3	5	4	6	2	

The cost matrix table associated with problem P_1 is shown in table 4.3.

Table 4.3: (Representation of costs of BCMIBTP through 1st mode of transportation)

Destinations

		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	10	9	12	7	8	7
	S_2	11	10	14	14	12	8
	S_3	8	6	10	10	13	9
	$b_j \rightarrow$	3	5	4	6	2	

On applying any of the cost-minimizing methods [8, 9] on the problem is shown in table 4.3, we have the solution $Y_{11} = \{y_{311}, y_{321}, y_{231}, y_{141}, y_{251}\}$ and the associated cost and time of transportation are $C_{11} = 47\ and\ T_{11} = 6$ respectively. The cost matrix table associated to problem P_2 is shown in table 4.4

Table 4.4: (Representation of Costs of BCMIBTP through 2nd mode of transportation)

Destinations

		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	12	10	14	8	7	7
	S_2	9	12	13	16	13	8
	S_3	7	5	10	12	11	9
	$b_j \rightarrow$	3	5	4	6	2	

Similarly, the cost-minimizing solution corresponding to the problem shown in table 4.4 is $Y_{12} = \{y_{212}, y_{322}, y_{332}, y_{142}, y_{252}\}$ and associated cost and time of transportation are $C_{12} = 45$ and $T_{12} = 8$ respectively.

Now, on applying step 3, we have $Y_1 = \{y_{212}, y_{322}, y_{332}, y_{141}, y_{251}\}$ is the first solution vector of BCMIBTP and the associated cost and time of transportation are $C_1 = 43$ and $T_1 = 8$ respectively. Thus, the first efficient cost-time trade-off pair of BCMIBTP is (43, 8).

To obtain the next efficient cost-time trade-off pair, delete the cells from both the tables of problems P_1 and P_2 for which time is more than or equal to 8. We obtain the following reduced cost matrix tables 4.5 and 4.6.

Table 4.5: (Reduced cost matrix table of BCMIBTP through 1st mode of transportation)

		Destinations					
		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	10	9	12	7	8	7
	S_2	11	10	14	14	12	8
	S_3	8	6	-	10	-	9
	$b_j \rightarrow$	3	5	4	6	2	

Table 4.6: (Reduced cost matrix table of BCMIBTP through 2nd mode of transportation)

		Destinations					
		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Sources	S_1	12	10	14	8	7	7
	S_2	9	12	13	16	13	8
	S_3	7	5	-	12	11	9
	$b_j \rightarrow$	3	5	4	6	2	

The cost-minimizing solution for the cost reduced problem shown in table 4.5 is given by $Y_{21} = \{y_{311}, y_{321}, y_{231}, y_{141}, y_{251}\}$ and the associated cost and time of transportation are $C_{21} = 47$ and $T_{21} = 6$ respectively. Similarly, the cost minimizing solution for the cost reduced problem shown in the table 4.6 is given by $Y_{22} = \{y_{312}, y_{322}, y_{232}, y_{142}, y_{252}\}$ and associated cost and time of transportation are $C_{22} = 46$ $andT_{22} = 7$ respectively.

Now, on applying step 3, we have the second efficient solution vector of BCMIBTP $Y_2 = \{y_{312}, y_{322}, y_{32}, y_{141}, y_{251}\}$. The associated cost and time of transportation are $C_2 = 44$ and $T_2 = 7$. Thus, the second efficient cost-time trade-off pair of BCMIBTP is (44, 7).Continuing in the same way, the next and last efficient solution vector of BCMIBTP is $Y_3 = \{y_{311}, y_{221}, y_{131}, y_{341}, y_{152}\}$ and the associated efficient cost-time trade-off pair is (47, 5). Thus, the all efficient cost-time trade-off pairs are (43, 8), (44, 7) and (47, 5).

5. Conclusion

In the present situation, cost and time of transportation have much influence on the profit in industries. It becomes a huge challenge for the businesspersons to reduce cost as well as the time of transportation for maximizing the revenue of the industries. In industries, normally products are supplied to destinations through different modes of transportation subject to availability of vehicles or the charges of transportation. The proposed method is quite useful in dealing cost-time trade-off relations in such type of problems and the proposed method provides all efficient cost-time trade-off pairs of BCMIBTP.

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