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Signed Edge Total Domination on Rooted Product Graphs

***C. Shobha Rani*[1], *S. Jeelani Begum*[2] and *G. Sankara Sekhar Raju*[3]**

[1,2]Department of Mathematics, Madanapalle Institute of Technology & Science
Madanapalle - 517325, India. [2]Email: sjbmaths@gmail.com
[3]Department of Mathematics, JNTUA College of Engineering,
Pulivendula - 516390, India. E-mail: rajugss@yahoo.com
[1]Corresponding author. E-mail: charapallishobha@gmail.com

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Abstract. Let G be a rooted product graph of path with a cycle graph with the vertex set V and the edge set E. Here P_n be a Path with n vertices and $C_m (m \geq 3)$ be a cycle with a sequence of n rooted graphs $C_{m1}, C_{m2}, C_{m3}, ---, C_{mn}$. We call $P_n(C_m)$ the rooted product of P_n by C_m and it is denoted by $P_n \circ C_m$. Every ith vertex of P_n is merging with any one vertex in every ith copy of C_m. In this paper we discuss some results on rooted product graph of path with a cycle graph.

Keywords: Signed edge total domination; Rooted product graph

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1. Introduction

Graph theory is one of the important branch in mathematics. Its applications are many like Engineering communications, Computer networking and etc. The rooted product graphs are usually used in internet networking for connecting internet to one network to other networks. Regularly product of graphs used in discrete mathematics.

In 1978, Godsil and McKay [2] introduced a new product on two graphs G_1 and G_2, called rooted product denoted by $G_1 \circ G_2$. In 1998, Haynes et al. [4] have studied about domination in graphs. Mitchell and Hedetniemi [7] and Xu [14] have worked on edge domination parameter. In 2014, Sinha et al. [9] have studied about 2-tuple domination problem on trapezoid graphs. In [8, 10] have studied about total domination related parameters and in [1, 13] have found some results on signed edge domination parameters. Velammal and Arumugam [11] have studied about total edge domination in graphs. In 2004, Henning [3] have studied about signed total domination in graphs. In 2012, Xia, Wei and Xu Chunlei [12] have studied about signed edge total domination numbers of two classes of graphs. Further we studied about rooted product graphs in [5, 6].

2. Results

Theorem 2.1. The signed edge total domination number of $G = P_n \circ C_m$ is

$\gamma'_{st}(G) = n(m+1) - 2\left\lceil \frac{n}{2} \right\rceil - 1$.

Proof: Let $G = P_n o C_m$ be a rooted product graph and m=3k or 3k+1 or 3k+2. Where k is a natural number set. We define a signed edge total dominating function $f : E \to \{-1,1\}$ as follows:

$$f(e) = \begin{cases} -1, \text{for} \left\lceil \frac{n}{2} \right\rceil \text{edges of } P_n \text{ in G,} \\ +1, \text{otherwise.} \end{cases}$$

Then by the definition of the function.

$f(e_1) = -1, f(e_2) = +1, f(e_3) = -1, f(e_4) = +1, ----.$

$f(h_{ij}) = 1,\ h_{ij} \in C_m.$

Case 1: If $e_i \in P_n$, where $i = 1, 2, ---, (n-1)$. Let $f(e_i) = +1$ then

If $\text{adj}(e_i) = 5$ then $\sum_{e \in N(e_i)} f(e) = (-1) + [1+1+1+1] = 3$.

If $\text{adj}(e_i) = 6$ then $\sum_{e \in N(e_i)} f(e) = (-1) + (-1) + [1+1+1+1] = 2$.

Let $f(e_i) = -1$ then

If $\text{adj}(e_i) = 5$ then $\sum_{e \in N(e_i)} f(e) = 1 + [1+1+1+1] = 5$.

If $\text{adj}(e_i) = 6$ then $\sum_{e \in N(e_i)} f(e) = 1 + 1 + [1+1+1+1] = 6$.

Case 2: If $h_{ij} \in C_m; i = 1, 2, ---, n; j = 1, 2, 3, ---, m$.

Subcase 1: Suppose $\text{adj}(h_{ij}) = 2$, $N(h_{ij})$, j=1,2,3,---,m there are no edges of P_n and two edges of C_m and there are two edges which are drawn from the vertices u_{ij} and $u_{i(j+1)}$ of C_m. Therefore $\sum_{e \in N(h_{ij})} f(e) = 1 + 1 = 2$.

Subcase 2: Suppose $\text{adj}(h_{ij}) = 3$, $N(h_{ij})$, j=1,2,3,---,m there are two edges of C_m, one edge of P_n and there is an edge which are drawn from the vertices u_{ij},i=1,2,---,n;j=1 or (m-1) and $v_i, i = 1 \text{ or } n$.

Therefore $\sum_{e \in N(h_{ij})} f(e) = \begin{cases} 1+1+1 = 3, \text{if } -1 \notin N(h_{ij}) \\ 1-1+1 = 1, \text{if } -1 \in N(h_{ij}) \end{cases}$.

Subcase 3: Suppose $\text{adj}(h_{ij}) = 4$, $N(h_{ij})$, j=1,2,3,---,m there are two edges of C_m, two edges of P_n and there is an edge which are drawn from the vertices u_{ij},i=1,2,---,n;j=1 or (m-1) and $v_i, i = 1 \text{ or } n$. Therefore $\sum_{e \in N(h_{ij})} f(e) = 1 - 1 + 1 + 1 = 2$.

Therefore from the above cases, we get $\sum_{e\in E(G)} f(e)\geq 1$. This implies f is a signed edge total dominating function. Now the minimality check for f. Define another function $g:E\rightarrow\{-1,1\}$ by

$$g(e)=\begin{cases}-1,\text{for}\left\lceil\frac{n}{2}\right\rceil\text{edges of } P_n \text{ in G},\\ -1,\text{if e} = \text{h}_k\in E \text{ for some k},\\ +1,\text{otherwise.}\end{cases}$$

Since strict equality not holds at an edge $\text{h}_k\in E$, it follows that $g<f$.

Case 1: If $e_i\in P_{\text{n}}$, where $i=1,2,---,(n-1)$.

Sub case 1: Let $\text{h}_k\in N(e_i)$.

Then if $\text{k}=i$ or $\text{i}+1,\text{if i}\neq(n-1)$ and k=1 or (n-1), if i=(n-1).

If $\text{adj}(e_i)=5$ then $\sum_{e\in N(e_i)} g(e)=\begin{cases}(-1)+[-1+1+1+1]=1,\text{ if g(e}_i)=+1\\ 1+[-1+1+1+1]=3,\text{ if g(e}_i)=-1\end{cases}$.

If $\text{adj}(e_i)=6$ then $\sum_{e\in N(e_i)} g(e)=\begin{cases}(-1)+(-1)+[-1+1+1+1]=0,\text{if g(e}_i)=+1\\ 1+1+[-1+1+1+1]=4,\text{if g(e}_i)=-1\end{cases}$.

Sub case 2: Let $\text{h}_k\notin N(e_i)$.

If $\text{adj}(e_i)=5$ then $\sum_{e\in N(e_i)} g(e)=\begin{cases}1+[1+1+1+1]=5,\text{ if g(e}_i)=-1\\ -1+[1+1+1+1]=4,\text{ if g(e}_i)=+1\end{cases}$.

If $\text{adj}(e_i)=6$ then $\sum_{e\in N(e_i)} g(e)=\begin{cases}1+1+[1+1+1+1]=6,\text{ if g(e}_i)=-1\\ -1-1+[1+1+1+1]=2,\text{ if g(e}_i)=+1\end{cases}$.

Case 2: If $h_{ij}\in C_m; i=1,2,---,n; j=1,2,3,---,m$.

Subcase 1: Suppose $\text{adj}(h_{ij})=2$, $\text{N}(h_{ij})$, j=1,2,3,---,m there are no edges of P_n and two edges of C_m and there are two edges which are drawn from the vertices u_ij and $\text{u}_{\text{i(j+1)}}$ of C_m. Therefore $\sum_{e\in N(h_{ij})} g(e)=\begin{cases}-1+1=0,\text{ if h}_k\in N(h_{ij})\\ 1+1=2,\text{ if h}_k\notin N(h_{ij})\end{cases}$.

Subcase 2: Suppose $\text{adj}(h_{ij})=3$, $\text{N}(h_{ij})$, j=1,2,3,---,m there are two edges of C_m, one edge of P_n and there is an edge which are drawn from the vertices u_{ij},i=1,2,---,n;j=1 or (m-1) and $\text{v}_i, i=1 or\, n$.

Let $\mathrm{h}_k \in \mathrm{N}(\mathrm{h}_{\mathrm{ij}})$ then $\sum_{e\in N(h_{ij})} f(e)=\begin{cases}-1+1+1=1, \text{if } +1\in N(h_{ij})\\ -1-1+1=-1, \text{if } -1\in N(h_{ij})\end{cases}$

Let $\mathrm{h}_k \notin \mathrm{N}(\mathrm{h}_{\mathrm{ij}})$ then $\sum_{e\in N(h_{ij})} g(e)=\begin{cases}1+1+1=3, \text{if } +1\in N(h_{ij})\\ 1-1+1=1, \text{if } -1\in N(h_{ij})\end{cases}$.

Subcase 3: Suppose $\mathrm{adj}(h_{ij})=4$, $\mathrm{N}(h_{ij})$, j=1,2,3,---,m there are two edges of C_{m}, two edges of P_{n} and there is an edge which are drawn from the vertices u_{ij}, i=1,2,---,n; j=1 or (m-1) and $\mathrm{v}_{\mathrm{i}}, \mathrm{i}=1 \text{ or } \mathrm{n}$.

Therefore $\sum_{e\in N(h_{ij})} g(e)=\begin{cases}1-1-1+1=0, \text{ if } \mathrm{h}_k \in N(h_{ij})\\ 1-1+1+1=2, \text{ if } \mathrm{h}_k \notin N(h_{ij})\end{cases}$.

Therefore from the above cases, we get $\sum_{e\in E(G)} g(e)<1, \text{for some } \mathrm{e}\in \mathrm{E(G)}$. This implies g is not a signed edge total dominating function. That is f is a minimal signed edge total dominating function. Now signed edge total domination number is

$$\sum_{e\in E(G)} f(e)=\left\lceil\frac{n}{2}\right\rceil(-1)+\left(n-1-\left\lceil\frac{n}{2}\right\rceil\right)(+1)+nm=n(m+1)-2\left\lceil\frac{n}{2}\right\rceil-1.$$

3. Conclusion

In this paper we obtain some results related to signed edge total domination of rooted product graphs and this work gives the scope for an extensive study of various inverse domination parameters of these graphs.

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