Annals of Pure and Applied Mathematics
Vol. 17, No. 1, 2018, 29-33
ISSN: 2279-087X (P), 2279-0888(online)
Published on 17 April 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v17n1a4

Annals of
Pure and Applied Mathematics

Edge Co-PI Indices of Special Graphs

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Received 16 March 2018; accepted 16 April 2018

Abstract. The edge Co-PI index of a graph G, denoted by $Co - PI_e(G)$, is defined as $Co - PI_e(G) = \sum_{e=uv\in E(G)} \left| m_u^G(e) - m_v^G(e) \right|$, where $m_u(e)$ denotes the number of edges of G whose distance to the vertex u is less than the distance to the vertex v. In this paper, the upper bounds for the edge Co-PI indices of Corona product product of two connected graphs is obtained. Finally, we compute the edge Co-PI indices of Tetrameric 1, 3-Adamantane.

Keywords: Co-PI index; corona graph

AMS Mathematics Subject Classification (2010): 05C12, 05C76

1. Introduction

All the graphs considered in this paper are connected and simple. A vertex $x \in V(G)$ is said to be *equidistant* from the edge $e = uv$ of G if $d_G(u, x) = d_G(v, x)$, where $d_G(u, x)$ denotes the distance between u and x in G. The degree of the vertex u in G is denoted by $d_G(u)$.

For an edge $uv = e \in E(G)$, the number of vertices of G whose distance to the vertex u is smaller than the distance to the vertex v in G is denoted by $n^G{}_u(e)$; analogously, $n^G{}_v(e)$ is the number of vertices of G whose distance to the vertex v in G is smaller than the distance to the vertex u; the vertices equidistant from both the ends of the edge $e = uv$ are not counted.

Similarly, $m_u(e)$ denotes the number of edges of G whose distance to the vertex u is less than the distance to the vertex v.

The vertex PI index of G, denoted by PI(G), is defined as $PI(G) = \sum_{e=uv\in E(G)} (n_u^G(e) + n_v^G(e))$. The Co- PI index of G, denoted by Co - PI(G), is defined as $Co - PI(G) = \sum_{e=uv\in E(G)} \left| n_u^G(e) - n_v^G(e) \right|$.

The edge PI index of G, denoted by $PI_e(G)$, is defined as $PI_e(G) = \sum_{e=uv\in E(G)} (m_u^G(e) + m_v^G(e))$.

The edge Co-PI index of G, denoted by Co-$PI_e(G)$, is defined as

$$Co - PI_e(G) = \sum_{e=uv\in E(G)} \left| m_u^G(e) - m_v^G(e) \right|.$$

The *PI* index of the graph G is a topological index related to equidistant vertices. Another topological index of G related to distance of G is the Wiener index of G, first introduced by Wiener, see [20]. Khadikar, Karmarkar and Agrawal [9] first introduced edge Padmakar-Ivan index of graphs and they investigated the chemical applications of the Padmakar-Ivan index. The mathematical properties of the PI_v and its applications in chemistry and nanoscience are well studied by Ashrafi and Loghman [2, 3], Ashrafi and Rezaei [4], Deng, Chen and Zhang [5], Khadikar [8], Klavzar [10] and Yousefi-Azari, Manoochehrian and Ashrafi [19]. The vertex PI indices of the tensor and strong products of graphs are studied in [14, 16]. In [11, 18, 12], the PI indices of bridge graphs and chain graphs are discussed. The properties of the edge Co-PI indices of graphs are discussed in [1]. In this paper, the upper bounds for the edge Co-PI indices of corona product and Tetrameric 1,3-Adamantane are obtained.

2. Corona product

Let G and H be two graphs. The *corona product* $G \circ H$, is obtained by taking one copy of G and $|V(G)|$ copies of H; and by joining each vertex of the i -th copy of H to the i -th vertex of G, where $1 \le i \le |V(G)|$, see Figure 1. For our convenience, we partition the edge set of $G \circ H$ into three sets, $E_1 = \{e \in E(G \circ H) / e \in E(H_i), 1 \le i \le n\}$, $E_2 = \{e \in E(G \circ H) / e \in E(G)\}$ *and* $E_3 = \{e \in E(G \circ H) / e = uv, u \in V(H_i), 1 \le i \le n, v \in V(G)\}$.

It is easy to see that E_1, E_2 and E_3 are partition of the edge set of $G \circ H$ and also $|E_1| = |V(G)||E(H)|, |E_2| = |E(G)|$ *and* $|E_3| = |V(G)||V(H)|$.

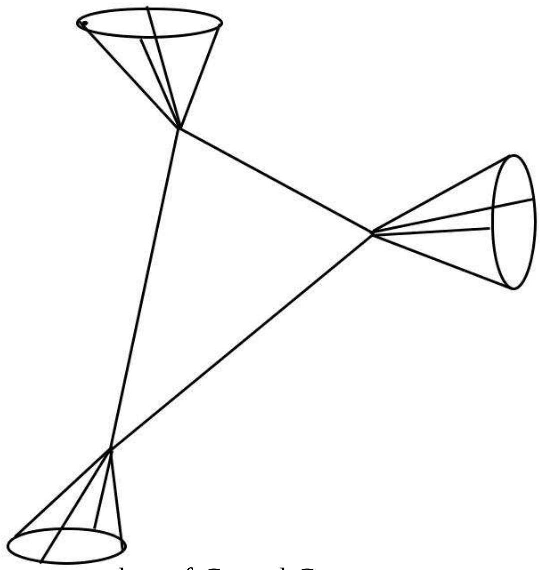

Figure 1: Corona product of C_3 and C_4

Theorem 2.1. *Let G be connected graph of order n and size p. If H is a triangle free and r - regular graph of order m and size q, then* $Co - PI_e(G \circ H) \le Co - PI_e(G) + n(Co - PI_e(H)) + (m + q)Co - PI(G) + nm(2r - p - n(m + q) + 1)$.

Proof: We partition the edges of $G \circ H$ into three sets E_1, E_2 and E_3 defined above.

First we compute $\sum_{e=uv \in E_1} \left| m_u^{G \circ H}(e) - m_v^{G \circ H}(e) \right|$.

Let $e = uv \in E_1$.

Then from the structure of $G \circ H$, we have $m_u^{G \circ H}(e) = m_u^G(e) + (m+q) n_u^G(e)$ and $m_v^{G \circ H}(e) = m_v^G(e) + (m+q) n_v^G(e)$

$$\sum_{e=uv\in E_1}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right| = \sum_{e=uv\in E(G)}\left|(m_u^G(e)+(m+q)n_u^{G\circ H}(e))\right|$$

$$= \sum_{e=uv\in E(G)}\left|((m_u^G(e)-m_v^G(e))+(m+q)(n_u^{G\circ H}(e))-n_v^{G\circ H}(e))\right|$$

$$-m_v^G(e)+(m+q)n_v^{G\circ H}(e))$$

$$\leq \sum_{e=uv\in E(G)}\left|m_u^G(e)-m_v^G(e)\right|+(m+q)\sum_{e=uv\in E(G)}\left|n_u^G(e)-n_v^G(e)\right|.$$

=(Co - PI_e(G)+(m+q)(Co -PI(G).

Next we compute $\sum_{e=uv\in E_2}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right|.$

Let $e = uv \in E_2$. Then from the structure of $G\circ H$, we have

$$m_u^{G\circ H}(e)=m_u^H(e)+1 \, and \, m_v^{G\circ H}(e)=m_v^H(e)+1.$$

$$\sum_{e=uv\in E_2}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right| = \sum_{i=1}^{n}\sum_{e=uv\in E(G)}\left|(m_u^H(e)+1)-(m_v^H(e)+1)\right|$$

$$= n\sum_{e=uv\in E(H)}\left|m_u^G(e)-m_v^G(e)\right|.$$

$$= n(\text{Co - PI}_e(H)).$$

Finally, we compute $\sum_{e=uv\in E_3}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right|.$

Let $e = uv \in E_3$. Then from the structure of $G\circ H$, we have

$$m_u^{G\circ H}(e)=d_H(u) \, and \, m_v^{G\circ H}(e)=\left|E(G\circ H)\right|-(d_H(u)+1)$$

$$\sum_{e=uv\in E_3}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right| = \sum_{u\in V(H)}\sum_{v\in V(G)}\left|d_H(u)-(\left|E(G\circ H)\right|-(d_H(u)+1))\right|$$

$$= \sum_{u\in V(H)}\sum_{v\in V(G)}(r-p-n(m+q)+r+1)$$

$$\leq nm(2r-p-n(m+q)+1).$$

Now we shall obtain the $Co - PI_e(G\circ H)$.

$$\text{Co - PI}_e(G\circ H) = \sum_{e=uv\in E_1}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right| + \sum_{e=uv\in E_2}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right|$$

$$+ \sum_{e=uv\in E_3}\left|m_u^{G\circ H}(e)-m_v^{G\circ H}(e)\right|.$$

$\leq Co - PI_e(G) + n(Co - PI_e(H)) + (m + q)Co - PI(G) + nm(2r - p - n(m + q) + 1).$

3. Edge Co-PI index of Tetrameric 1,3-Adamantane

From the structure of the graph tetrameric 1, 3-adamantane $T\,A[n]$, the number of vertices and edges are $10n$ and $13n - 1$, respectively, see Figure 2.

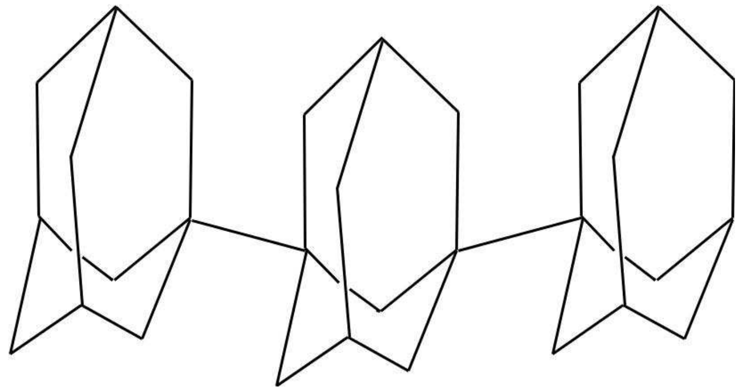

Figure 2: The tetrameric 1,3-adamantane (TA[3])

Theorem 3.1. *The edge Co-PI index of $TA[n]$ is $Co-PI_e(TA[n]) \le 18n$.*

Proof: From the structure of the graph $TA[n]$, we have the following cases of edges.

If $e=u_iv_i$ then $m_{u_i}^G(e) = 13i - 1 \, and \, m_{v_i}^G(e) = 13(n-i) - 1$.

If $e=uv=\{x_1x_2, x_5u_k, x_6x_7\}$, then $m_u^G(e) = 6 + 13(n-k) \, and \, m_v^G(e) = 3 + 13(k-1)$.

If $e=uv=\{x_1x_4, v_{k-1}x_5, x_7x_8\}$, then $m_u^G(e) = 6 + 13(k-1) \, and \, m_v^G(e) = 3 + 13(n-k)$.

If $e=uv=\{x_2v_{k-1}, x_3x_7, x_4u_k\}$, then $m_u^G(e) = 3 \, and \, m_v^G(e) = 6 + 13(n-k) + 13(k-1)$.

If $e=uv=\{x_1x_3, u_kx_8, v_{k-1}x_6\}$, then $m_u^G(e) = 6 + 13(n-k) + 13(k-1) \, and \, m_v^G(e) = 3$.

$$Hence, Co-PI_e(G) = \sum_{e=uv\in E(G)} \left| m_u^G(e) - m_v^G(e) \right|.$$

$$= \sum_{i=1}^{n-1} \left| m_u^G(e) - m_v^G(e) \right| + 3\sum_{k=1}^{n} \left| m_u^G(e) - m_v^G(e) \right| + 3\sum_{i=1}^{n} \left| m_u^G(e) - m_v^G(e) \right|$$

$$+ 3\sum_{k=1}^{n} \left| m_u^G(e) - m_v^G(e) \right| + 3\sum_{i=1}^{n} \left| m_u^G(e) - m_v^G(e) \right|$$

$$= \sum_{k=1}^{n-1} \left| (13i-1) - (13(n-i)-1) \right| + 3\sum_{i=1}^{n} \left| (6+13(n-k)) - (3+13(k-1)) \right|$$

$$+ 3\sum_{i=1}^{n} \left| (6+13(k-1)) - (3+13(n-k)) \right|$$

$$+ 3\sum_{k=1}^{n} \left| 3 - (6+13(n-k)+13(k-1)) \right| + 3\sum_{k=1}^{n} \left| 6+13(n-k)+13(k-1) - 3 \right|.$$

$\le 18n$.

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