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Second Order Hybrid Fuzzy Fractional Differential Equations by Runge Kutta 6th Order Fehlberg Method

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Abstract. In this paper we study numerical methods for second order hybrid fuzzy fractional differential equations. We solve the hybrid fuzzy fractional differential equations with a fuzzy initial condition by using variational iteration method. We consider a second order differential equation with fractional values and we compared the results with their exact solutions in order to demonstrate the validity and applicability of the method. We further give the definition of the Degree of Sub element hood of hybrid fuzzy fractional differential equations with examples.

Keywords: Hybrid fuzzy fractional differential equations, degree of sub element hood

AMS Mathematics Subject Classification (2010): 65L40, 65L70

1. Introduction

With the rapid development of linear and nonlinear science, many different methods such as the variational iteration method (VIM) [1] were proposed to solve fuzzy differential equations. Fuzzy initial value problems for fractional differential equations have been considered by some authors recently [2,3]. As another development, hybrid systems are dynamical systems that progress continuously in time but have formatting changes called modes at a sequence of discrete times. Some recent papers about hybrid systems include [4,7]. When the continuous time dynamics of a hybrid system comes from fuzzy fractional differential equations the system is called a hybrid fuzzy fractional differential system or a hybrid fuzzy fractional differential equation.

This paper is organized as follows. In Section 2, we provide some background on fuzzy fractional differential equations and hybrid fuzzy fractional differential equations. In Section 3 we discuss the numerical solution of Second order hybrid fuzzy fractional differential equations by Runge Kutta 6th order Fehlberg method. The method given uses piecewise application of a numerical method for fuzzy fractional differential equations. In Section 4, as an example, we numerically solve the Degree of Sub element hood of hybrid fuzzy fractional differential equations. The objective of the present paper is to extend the application of the variational iteration method, to provide approximate solutions for fuzzy initial value problems of differential equations of fractional order, and to make comparison with that obtained by an exact fuzzy solution.

2. Hybrid fuzzy fractional differential equations

2.1. Preliminaries

In this section the most basic notations used in hybrid fuzzy fractional differential equations are introduced.

Let us consider the following fractional differential equation:

$$ {}_cD_a^{\beta} x(t) = f(t, x(t), \lambda_k(x_k)), \qquad t \in [t_k, t_{k+1}] \tag{1} $$
$$ x(t_k) = x_k $$

where, $0 \le t_0 \le t_1 \le \cdots \le t_k \to \infty$

$$ f \in C[R^+ \times E \times E, E], \lambda_k \in C[E, E] $$

We can also represent a fuzzy numbers x ∈ E by a pair of functions

$$ {}_cD_a^{\beta}\ x(t) = {}_cD_a^{\beta}[\underline{x}(t;r), \overline{x}(t;r)] $$
$$ = [\, {}_cD_a^{\beta}\underline{x}(t), {}_cD_a^{\beta}\overline{x}(t)\ ] $$

Using a representation of fuzzy numbers we may represent x ∈ E by a pair of functions $(\underline{x}(r), \overline{x}(r)), 0 \le r \le 1$, such that:

1. $\underline{x}(r)$ is bounded, left continuous and non decreasing,
2. $\overline{x}(r)$ is bounded, left continuous and non increasing and
3. $\underline{x}(r) \le \overline{x}(r),\ 0 \le r \le 1$

Therefore, we may replace (1) by an equivalent system equation (2):

$$ \begin{cases} {}_cD_a^{\beta}\underline{x}(t) = \underline{f}(t, x, \lambda_k(x_k) \equiv F_k(t, \underline{x}, \overline{x}), \underline{x}(t_k) = \underline{x_k} \\ {}_cD_a^{\beta}\overline{x}(t) = \overline{f}(t, x, \lambda_k(x_k) \equiv G_k(t, \underline{x}, \overline{x}), \overline{x}(t_k) = \overline{x_k} \end{cases} \tag{2} $$

This possesses a unique solution $(\underline{x}, \overline{x})$, which is a fuzzy function. That is for each t, the pair $[\underline{x}(t;r), \overline{x}(t;r)]$ is a fuzzy number, where $\underline{x}(t;r), \overline{x}(t;r)$ are respectively the solutions of the parametric form given by Equation (3):

$$ \begin{cases} {}_cD_a^{\beta}\underline{x}(t) = F_k(t, \underline{x}(t;r), \overline{x}(t;r)), \underline{x}(t_k;r) = \underline{x_k}(r) \\ {}_cD_a^{\beta}\overline{x}(t) = G_k(t, \underline{x}(t;r), \overline{x}(t;r)), \overline{x}(t_k;r) = \overline{x_k}(r) \\ \qquad \text{for } r \in [0, 1] \end{cases} \tag{3} $$

3. The sixth order Runge Kutta Fehlberg method with harmonic mean

For a second order hybrid fuzzy fractional differential equation we develop the sixth order Runge Kutta Fehlberg method with harmonic mean when f and λ_k in (1) can be obtained via the Zadeh extension principle from:

$$ f \in [R^+ X R X R, R] \text{ and } \lambda_k \in C[R,R] $$

The Sixth Order Runge Kutta Fehlberg method for (1) is given by:

$$ (\underline{Y}_k(t;r), \overline{Y}_k(t;r) \equiv (\underline{x}_k(t;r), \overline{x}_k(t;r)), (\underline{y}_k(t;r), \overline{y}_k(t;r)) $$

where

$$\underline{k_1}(t_{k,n};y_{k,n}(r);z_{k,n}(r)) = \min\left\{\begin{array}{l} h_k f(t_{k,n},u,\lambda_k(u_k)) \\ \backslash u \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \\ u_k \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \end{array}\right\},$$

$$\underline{l_1}(t_{k,n};y_{k,n}(r);z_{k,n}(r)) = \min\left\{\begin{array}{l} h_k f(t_{k,n},u,\lambda_k(u_k)) \\ \backslash u \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \\ u_k \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \end{array}\right\},$$

$$\overline{k_1}(t_{k,n};y_{k,n}(r);z_{k,n}(r)) = \max\left\{\begin{array}{l} h_k f(t_{k,n},u,\lambda_k(u_k)) \\ \backslash u \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \\ u_k \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \end{array}\right\},$$

$$\overline{l_1}(t_{k,n};y_{k,n}(r);z_{k,n}(r)) = \max\left\{\begin{array}{l} h_k f(t_{k,n},u,\lambda_k(u_k)) \\ \backslash u \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \\ u_k \in \{[\underline{y}_{k,n}(r),\overline{y}_{k,n}(r)],[\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]\} \end{array}\right\},$$

$$\underline{k_2}(t_{k,n};y_{k,n}(r);z_{k,n}(r)) = \min\left\{\begin{array}{l} h_k f(t_{k,n}+\frac{1}{2}(h_k),u,\lambda_k(u_k)) \\ \backslash u \in \begin{bmatrix} \underline{\Phi}_{k_1}(t_{k,n},y_{k,n}) \\ \overline{\Phi}_{k_1}(t_{k,n},y_{k,n}) \end{bmatrix} \\ u_k \in [\underline{y}_{k,0}(r),\overline{y}_{k,0}(r)] \end{array}\right\},$$

Like we can arrange

$\underline{l_2}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{k_2}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{l_2}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$
$\underline{k_3}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\underline{l_3}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{k_3}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$,
$\overline{l_3}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\underline{k_4}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\underline{l_4}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$,
$\overline{k_4}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{l_4}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{k_5}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$,
$\overline{l_5}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$, $\overline{k_6}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$ & $\overline{l_6}(t_{k,n};y_{k,n}(r);z_{k,n}(r))$

$$\underline{\Phi}_{k_1}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \underline{f}(t_{k,n} + 1/4 * h, \underline{y}_{k,n}(r) + \underline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \underline{z}_{k,n}(r) + \underline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) * h)$$

$$\overline{\Phi}_{k_1}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \overline{f}(t_{k,n} + 1/4 * h, \overline{y}_{k,n}(r) + \overline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \overline{z}_{k,n}(r) + \overline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) * h)$$

$$\underline{\Phi}_{k_2}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \underline{f}(t_{k,n} + 3/8 * h, \underline{y}_{k,n}(r) + (3/32) * h * (\underline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 3 * \underline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \underline{z}_{k,n}(r) + (3/32) * h * (\underline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 3 * \underline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$$

$$\overline{\Phi}_{k_2}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \overline{f}(t_{k,n} + 3/8 * h, \overline{y}_{k,n}(r) + (3/32) * h * (\overline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 3 * \overline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \overline{z}_{k,n}(r) + (3/32) * h * (\overline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 3 * \overline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$$

$$\underline{\Phi}_{k_3}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \underline{f}(t_{k,n} + (12/13) * h, \underline{y}_{k,n}(r) + (12/2197) * h * (161 * \underline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 600 * \underline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 608\underline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \underline{z}_{k,n}(r) + (12/2197) * h * (161 * \underline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 600 * \underline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 608\underline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$$

$$\overline{\Phi}_{k_3}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \overline{f}(t_{k,n} + (12/13) * h, \overline{y}_{k,n}(r) + (12/2197) * h * (161 * \overline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 600 * \overline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 608\overline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \overline{z}_{k,n}(r) + (12/2197) * h * (161 * \overline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 600 * \overline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 608\overline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$$

$$\underline{\Phi}_{k_4}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \underline{f}(t_{k,n} + h, \underline{y}_{k,n}(r) + (1/4104) * h * (8341 * \underline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 32832 * \underline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 29440 * \underline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 845 * \underline{k}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \underline{z}_{k,n}(r) + (1/4104) * h * (8341 * \underline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 32832 * \underline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 29440 * \underline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 845 * \underline{l}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$$

$\overline{\Phi}_{k_4}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \overline{f}(t_{k,n} + h, \overline{y}_{k,n}(r)$
$+ (1/4104) * h * (8341 * \overline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$- 32832 * \overline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 29440 * \overline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$- 845 * \overline{k}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \overline{z}_{k,n}(r)$
$+ (1/4104) * h * (8341 * \overline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 32832 * \overline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$+ 29440 * \overline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - 845 * \overline{l}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$

$\underline{\Phi}_{k_5}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \underline{f}(t_{k,n} + (0.5)h, \underline{y}_{k,n}(r)$
$+ h * (-(8/27) * \underline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$+ 2 * \underline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - (3544/2565) * \underline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$+ (1859/4104) * \underline{k}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - (11/40) * \underline{k}_5(t_{k,n}, y_{k,n}(r), z_{k,n}(r)), \underline{z}_{k,n}(r)$
$+ h * (-(8/27) * \underline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 2 * \underline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$- (3544/2565) * \underline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + (1859/4104) * \underline{l}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$- (11/40) * \underline{l}_5(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$

$\overline{\Phi}_{k_5}(t_{k,n}, y_{k,n}(r), z_{k,n}(r)) = \overline{f}(t_{k,n} + (0.5)h, \overline{y}_{k,n}(r)$
$+ h * (-(8/27) * \overline{k}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$+ 2 * \overline{k}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - (3544/2565) * \overline{k}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r)$
$+ (1859/4104) * \overline{k}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r) - (11/40) * \overline{k}_5(t_{k,n}, y_{k,n}(r), z_{k,n}(r)),$
$\overline{z}_{k,n}(r) + h * (-(8/27) * \overline{l}_1(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + 2 * \overline{l}_2(t_{k,n}, y_{k,n}(r), z_{k,n}(r) -$
$(3544/2565) * \overline{l}_3(t_{k,n}, y_{k,n}(r), z_{k,n}(r) + (1859/4104) * \overline{l}_4(t_{k,n}, y_{k,n}(r), z_{k,n}(r) -$
$(11/40) * \overline{l}_5(t_{k,n}, y_{k,n}(r), z_{k,n}(r)))$

Next we define:

$S_k[t_{k,n,} \underline{y}_{k,n}(r), \overline{y}_{k,n}(r), \underline{z}_{k,n}(r), \overline{z}_{k,n}(r)] =$
$\frac{h}{5}\{(16/27)\underline{k}_1(t_{k,n}; y_{k,n}(r), z_{k,n}(r)) + (6656/2565)\underline{k}_3(t_{k,n}; y_{k,n}(r), z_{k,n}(r))$
$+ (28561/11286)\underline{k}_4(t_{k,n}; y_{k,n}(r), z_{k,n}(r)) - (9/10)\underline{k}_5(t_{k,n}; y_{k,n}(r), z_{k,n}(r)$
$+ (12/11)\underline{k}_6(t_{k,n}; y_{k,n}(r), z_{k,n}(r))\}$

$$T_k[t_{k,n,}\underline{y}_{k,n}(r),\overline{y}_{k,n}(r),\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]=$$

$$\frac{h}{5}\{(16/27)\overline{k}_1(t_{k,n};y_{k,n}(r),z_{k,n}(r))+(6656/2565)\overline{k}_3(t_{k,n};y_{k,n}(r),z_{k,n}(r))$$

$$+(28561/11286)\overline{k}_4(t_{k,n};y_{k,n}(r),z_{k,n}(r))-(9/10)\overline{k}_5(t_{k,n};y_{k,n}(r),z_{k,n}(r)$$

$$+(12/11)\overline{k}_6(t_{k,n};y_{k,n}(r),z_{k,n}(r))\}$$

The exact solution at $t_{k,n+1}$ is given by:

$$\begin{cases} F_{k.n+1}(r)=\underline{Y}_{k,n}(r)+S_k[t_{k,n,}\underline{y}_{k,n}(r),\overline{y}_{k,n}(r),\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)], \\ G_{k,n+1}(r)=\overline{Y}_{k,n}(r)+T_k[t_{k,n,}\underline{y}_{k,n}(r),\overline{y}_{k,n}(r),\underline{z}_{k,n}(r),\overline{z}_{k,n}(r)]. \end{cases} \quad (5)$$

3.1. Degree of sub element hood

Let X be a Universal, U be a set of parameters and let ($F_{k,n+1}$) and ($G_{k,n+1}$) are two fuzzy elements of X. Then the degree of sub element hood denoted by

$\mathfrak{S}$($F_{k,n+1}$, $G_{k,n+1}$) is defined as,

$$\mathfrak{S}(F_{k,n+1},G_{k,n+1})=\frac{1}{|(F_{k,n+1})|}\left\{|(F_{k,n+1})|-\sum\max\{0,\left(F_{k,n+1}\right)-\left(G_{k,n+1}\right)\}\right\}$$

Where $\left|F_{k,n+1}\right| = \sum_{e_j\in A}\exp(F_{k,n+1})$ and

$$\mathfrak{S}(G_{k,n+1},F_{k,n+1})=\frac{1}{|(G_{k,n+1})|}\left\{|(G_{k,n+1})|-\sum\max\{0,\left(G_{k,n+1}\right)-(F_{k,n+1})\}\right\}.$$

4. Numerical example

In this section, we present the example for solving hybrid fuzzy fractional differential equations.

Consider the following second order hybrid fuzzy fractional differential equation:

$${}_cD_a^{\beta}\ X(t)=Z \text{ and } {}_cD_a^{\beta} Z(t) = XZ^2\text{-}Y^2 \quad (6)$$

$$X(0) = X_0,$$

where β∈(0,1] and t > 0 and X_0 is any triangular fuzzy number.

This problem is a generalization of the following hybrid fuzzy fractional differential equation:

$${}_cD_a^{\beta}\ x(t) = z(t) = [\underline{z}(t;r),\overline{z}(t;r)]\ \&$$

$${}_cD_a^{\beta} z(t) = [\underline{x}(t;r),\overline{x}(t;r)] * [\underline{z}(t;r),\overline{z}(t;r)]^2\text{-}[\underline{y}(t;r),\overline{y}(t;r)]^2 \quad (7)$$

$$x(t) = x_0,$$

where β∈(0,1], t > 0, α is the step size and x_0 is a real number.

We can find the solution of the hybrid fractional fuzzy differential equation, by the method of Runge Kutta 4th order & Fehlberg 6th order Methods. We compared and generalized the error of the hybrid fractional fuzzy differential equation, also we illustrated the figure and in the table for this generalization by using Matlab.

S. No	T	RK 6th order (W)	RK 6th order (Z)
1	0	0	1.3
2	0.1	1.38252921	0.843479358
3	0.2	1.45043447	0.672041993
4	0.3	1.50232107	0.484592947
5	0.4	1.53679615	0.2823831
6	0.5	1.55270406	0.069581679
7	0.6	1.54940226	-0.14644294
8	0.7	1.52703428	-0.35553489
9	0.8	1.48672389	-0.54612954
10	0.9	1.43060781	-0.70745571
11	1.0	1.36166109	-0.8318705

Table 1: Numerical Solution of the example

$\mathfrak{S}(F_{k,n+1}, G_{k,n+1}) \cong 1$ & $\mathfrak{S}(G_{k,n+1}, F_{k,n+1}) \cong 0.8$

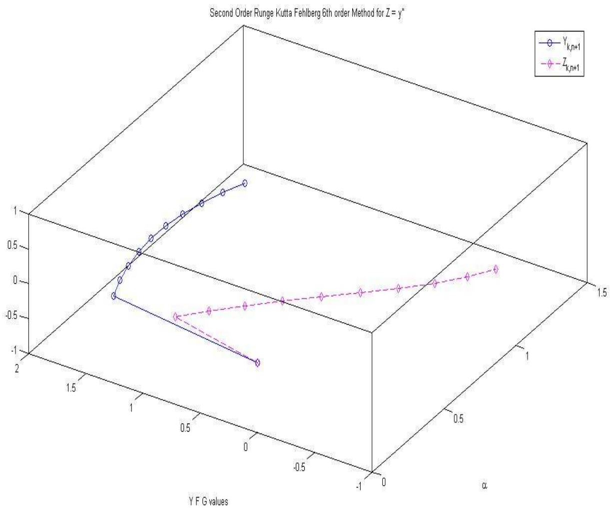

Figure 1: Graphical representation of the example

5. Conclusion

In this paper, we have studied a hybrid fuzzy fractional differential equation. Final results showed that the solution of hybrid fuzzy fractional differential equations approaches the solution of hybrid fuzzy differential equations as the fractional order approaches the integer order. The results of the study reveal that the proposed method with fuzzy fractional derivatives is efficient, accurate, and convenient for solving the hybrid fuzzy fractional differential equations.

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