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Future Population Projection of Bangladesh by Growth Rate Modeling Using Logistic Population Model

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Abstract. Population of Bangladesh has been predicted with the help of an ordinary differential equation model known as logistic population model which is parameterized by growth rate along with capacity human population of Bangladesh. We use fourth order Runge-Kutta scheme for the numerical solution of the non-autonomous and non-linear model where we incorporate the growth rate as a function of time. First we test the numerical method for Bangladesh population data (1991-2006) and we find our population projection which is very good fit with the actual data. Then we implement the logistic model that gives future population projection for Bangladesh during 1996 to 2035.

***Keywords*:** Growth rate, Logistic Population Model, Curve fitting, Vital coefficient.

AMS Mathematics Subject Classifications (2010): 35G30, 35G60.

1. Introduction

Population problem is one of the main problems in Bangladesh at the current time. Bangladesh is an overpopulated country and the growth in resources has not been keeping pace with the growth in population. So the increasing trend in population is a great threat to the nation. Kabir and Chowdhury [6] investigated the relationship between population growth and food production in Bangladesh. Recognizing the difficulty of feeding the growing population even with considerable increase in food production, they suggested giving priority to population policy for reduction in population.

In this situation, prediction of population is very essential for planning. Obaidullah [8] presented a model termed as '*Expo-linear Model*' which is claimed better than either an exponential or a linear model in describing population growth over time. But he agreed that there was a difficulty in his model in interpreting its parameter unlike those of exponential or linear one.

Mallick [7] studied the population trend in Bangladesh and commented that the possibility of zero population growth in the next hundred years was very unrealistic. Beekman [5] developed exponential growth models and models utilizing a Markov chain to reflect lower birth rates caused by rural-urban movement. Using these models, population of Bangladesh were projected to the years 1998, 2018 and 2038 respectively. Beekman [5] developed confidence intervals of population in Bangladesh for years 1986 and 2001. He also analyzed rural-urban shifts in population through Markov chains. Assuming a fixed percentage decrease in the subsequent annual growth rates, he calculated the required percentage reduction in growth rate from 2.70 in 1974 to 2.36 in 1981 as 0.02 by solving the equation $2.70(1-x)^7$=2.36. Using this reduction rate in 5 year blocks, population in 2001 was computed and compared with values in the confidence interval.

Rahman [9] used the formula $p_t = p_0 e^{(r+k)t}$ where $p_0 =$ current population; $p_t =$ population after time *t;* $r =$ growth rate and *k* is annual migration rate, for prediction of regional population.

In many cases, for small population one may use the discrete model as:

$$p_t = p_0 (1+r)^t \qquad \text{(1.1)}$$

where $p_0 =$ current population; $p_t =$ population after time *t;* $r =$ growth rate.

In [3], Hoque, Ahmed and Sarker used this discrete model with the modification of the growth rate which is not constant. They showed the population of Bangladesh as projected for the period 1976-2093 and obtained a parabolic profile; where the critical point obtained at $t = 2035$, it means that at $t = 2035$, $p'(t) = 0$ i.e. the growth rate become zero in the year 2035. After 2035 population decreases in the parabolic manner.

The continuous analogue of (1.1) is the Malthusian ODE model

$$\frac{dp}{dt} = ap \qquad \text{(1.2)}$$

i.e. $p(t) = p_0 e^{at}$ .... (1.3)

where a is a constant.

The Malthusian model is very simple and applicable for small population and therefore for large population it is preferable to use logistic ODE population model:

$$\frac{dp}{dt} = a\,p - b\,p^2 \qquad \text{(1.4)}$$

where a and b are called vital coefficients.

We use the *Logistic Population Model* which was first introduced in 1837 by the Dutch mathematical-biologist Verhulst to predict the future population of Bangladesh. If p_0 is the population at time t_0, then $p(t)$, the population at time t, satisfies the initial-value problem (IVP):

$$\frac{dp}{dt} = ap - b\,p^2 \qquad \text{……………………………………………………… (1.5)}$$
$$p(t_0) = p_0$$

It is easier to calculate the analytical solution for IVP (1.5) when the vital coefficients *a* and *b* are considered constants. But it is not so easy to calculate the analytical solution for $a(t)$ and $b(t)$ as functions of *t.* To compute the vital coefficients $a(t)$ and $b(t)$ as functions of *t*, it could be more convenient to use numerical methods based on some efficient algorithm. Therefore we are interested to study some well-understood numerical schemes to solve the logistic model where we could also calculate the vital coefficients $a(t)$ and $b(t)$ as functions of *t* based on some algorithm.

In the population prediction of Bangladesh based on a non-linear, non-autonomous ordinary differential equation model which is known as generalized Logistic Population Model and parameterized by growth rate along with capacity. In terms of carrying capacity Logistic Differential equation can also be defined as:

$$\frac{1}{p}\frac{dp}{dt} = a\left(1 - \frac{p}{k}\right) = R(t) \qquad \text{………………………………….…….. (1.6)}$$

where (1.6) represents the growth rate and k presents carrying capacity. Here the calculations are based on parameters characterizing growth rate and carrying capacity.

Now in this paper we also implement the generalized logistic population model where the growth rate R (t) is estimated by fitting a curve on the data in the past several years

2. The logistic population model

In this section we discuss in detail a model for population growth, the logistic model, which is more sophisticated than exponential growth. The logistic model, a slight modification of Malthus's model, is just such a model.

When the population gets extremely large though, Malthus's model can not be very accurate, since they do not reflect the fact that individual members are now competing with each other for the limited living space, natural resources food available. Thus, we must add a competition term to our linear differential equation. A suitable choice of a competition term is $-bp^2$, where b is a constant, since the statistical average of the number of encounters of two members per unit time is proportional to p^2. We consider, therefore the modified equation

$$\frac{dp}{dt} = ap - bp^2 \qquad \text{………...……………………………………….. (2.1)}$$

This equation is known as the logistic law of population growth and the numbers ***a***, ***b*** are called the vital coefficients of the population. It was first introduced in 1837 by the Dutch mathematical-biologist Verhulst. Now, the constant ***b***, in general, will be very small compared to ***a*,** so that if p is not too large then the term $-bp^2$ will be negligible compared to ap and the population will grow exponentially. However, when p is very large, the term $-bp^2$ is no longer negligible, and thus serves to slow down the rapid rate of increase of the population. Needless to say, the more industrialized a nation is, the more living space it has, and the more food it has, the smaller the coefficient b is.

Let us now use the logistic equation to predict the future growth of an isolated population. If p_0 is the population at time t_0, then $p(t)$, the population at time t, satisfies the initial-value problem

$$\frac{dp}{dt}=ap-b\,p^2 \qquad p(t_0)=p_0 \qquad \text{……(2.2)}$$

3. Analytical solution of logistic population model

The logistic equation can be solved by separation of variables. From equation (2.2), we have

$$\frac{dp}{dt}=ap-b\,p^2 \qquad \text{……(2.3)}$$

Integrating on both sides of this equation then we get

$$\int\frac{dp}{ap-bp^2}=\int dt$$

$$\therefore \frac{1}{a}\left[\log p-\log(a-bp)\right]=t+c \qquad \text{……(2.4)}$$

Using $t=t_0$ and $p=p_0$ $\quad \therefore c=\frac{1}{a}[\log p_0-\log(a-bp_o)]-t_0$

Now substituting the value of c into equation (2.4) and simplifying, we have

$$p(t)=\frac{ap_0}{bp_0+(a-bp_0)e^{-a(t-t_0)}} \qquad \text{……(2.5)}$$

This is the required analytical solution of the logistic equation.

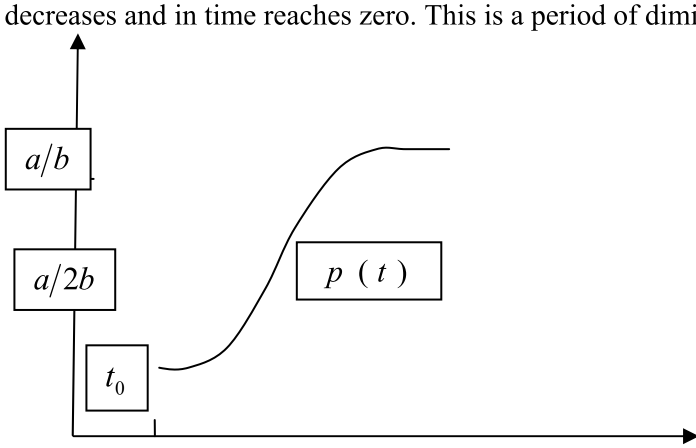
Let us now examine Equation (2.5) to see what kind of population it predicts. We observe that as $t\to\infty$ then $p(t)\to\frac{ap_0}{bp_0}=\frac{a}{b}$

Thus regardless of its initial value, the population always approaches the limiting value a/b. Next, we observed that $p(t)$ is monotonically increasing function of the time if $0\langle\, p_0\,\langle\, a/b$.

Moreover, since

$$\frac{d^2p}{dt^2} = a\frac{dp}{dt} - 2bp\frac{dp}{dt} = (a-2bp)\frac{dp}{dt} = (a-2bp)p(a-bp)$$

We see that dp/dt is increasing if $p(t) \langle\ a/b$, and that dp/dt is decreasing if $p(t) \rangle\ a/2b$. Hence if $p_0 \ \langle\ a/2b$, the graph of $p(t)$ must have the form given in the figure. Such a curve is called a logistic, or S – shaped curve. From its shape we conclude that the time period before the population reaches the half its limiting value is a period of accelerated growth. After this point, the rate of growth decreases and in time reaches zero. This is a period of diminishing growth.



4. **Analytical and numerical results**

In here we present some results of the numerical and analytical solution of the logistic population model. A fourth order Rung-Kutta scheme has been used to solve logistic population model as an IVP. For this we estimate the coefficients a and b in the logistic equation and calculate the values of a and b from data. In this computation, the vital coefficients a and b are assumed as constants. We also try to present some results with a and b as function of t.

4.1. Calculation of vital coefficients

We have the analytical solution of the logistic equation as:

$$p(t) = \frac{ap_0}{bp_0 + (a-bp_0)e^{-a(t-t_0)}} \quad \text{.......................................(3.1)}$$

Let $p_0 =$ population at time $t = t_0$, $p_1 =$ population at time $t = t_1$ and also let $t_1 - t_0 = \Delta t$,

In this case, calculation of the values of a and b is performed in order to predict the population of Bangladesh.

For this we assume $p_0 =$ population of Bangladesh in 1991= 112,200,000

$p_1 =$ population of Bangladesh in 1996 = 122,100,000

$p_2 =$ population of Bangladesh in 2001 = 131,100,000

and k= 176771641.8, b= 2.84000×10^{-10} a=0.05

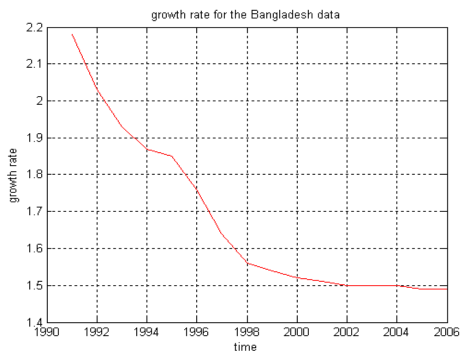
4.1. Change in Population Size at the Terminal Point, 1981- 2006

(Population in million)

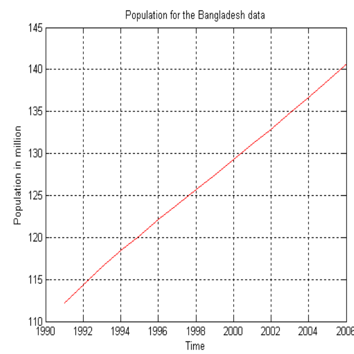
Year	Population (1^{st} January)	Population (1^{st} July)	No. of Birth	No. of Death	Natural Growth (Birth-Death)	Natural Growth Rate(%)
1981	89.9	90.4	3.098	1.038	2.06	2.28
1982	91.4	92.3	3.189	1.107	2.08	2.25
1983	93.3	94.3	3.280	1.163	2.12	2.25
1984	95.3	96.3	3.335	1.182	2.15	2.23
1985	97.4	98.4	3.392	1.183	2.21	2.25
1986	99.5	100.5	3.448	1.183	2.27	2.26
1987	101.7	102.8	3.414	1.173	2.24	2.18
1988	103.9	105.0	3.477	1.179	2.30	2.19
1989	106.2	107.4	3.531	1.196	2.34	2.18
1990	108.6	109.8	3.559	1.106	2.45	2.23
1991	111.5	112.2	3.561	1.110	2.45	2.18
1992	113.3	114.4	3.455	1.139	2.32	2.03
1993	115.5	116.5	3.350	1.100	2.25	1.93
1994	117.5	118.4	3.289	1.067	2.22	1.87
1995	119.3	120.2	3.228	1.007	2.22	1.85
1996	121.2	122.1	3.143	0.989	2.15	1.76
1997	123.0	123.9	2.746	0.719	2.03	1.64
1998	124.8	125.7	2.608	0.652	1.96	1.56
1999	126.6	127.5	2.542	0.649	1.89	1.48
2000	128.4	129.3	2.454	0.640	1.81	1.40
2001	130.0	131.1	2.439	0.638	1.80	1.40
2002	132.0	132.9	2.674	0.679	1.99	1.50
2003	133.9	134.8	2.814	0.783	2.01	1.50
2004	135.9	136.7	2.830	0.794	2.00	1.50
2005	137.8	138.6	2.879	0.823	2.07	1.49
2006	139.8	140.6	2.900	0.789	2.11	1.49

Source: "Report on sample vital registration system, 2005-2006"
Published on December – 2007, Bangladesh Statistical Bureau, Bangladesh.

Growth rate of Bangladesh



Population of Bangladesh



4.2. Fitting a logistic model to data

We repeat from the actual data of Bangladesh Statistical Bureau trough 2006, together with a fitted logistic curve. We will determine directly from the differential equation how to tell whether a given set of data is reasonably logistic and if so what parameters a and k will give a good fit. Our test case will be the Data of Bangladesh Statistical Bureau, first up to 1981, then up to 2006. To determine whether a given set of data can be modeled by the logistic differential equation we have to estimate values of the derivative $\frac{dp}{dt}$ from the data. We will do that by symmetric differences. The slope $\frac{dp}{dt}$ at a given census year t is approximated by the slope of the line joining the points 5 years earlier and 5 years later. For example, the growth rate $\frac{dp}{dt}$ in 2000 was approximately [p(2005)-p(1995)] / 10. We may rewrite the logistic equation in the form

$$\frac{dp}{dt} = ap(1-\frac{p}{k}) \quad \text{…………………………………………………………………………….(3.2)}$$

where $\frac{1}{p}\frac{dp}{dt} = a(1-\frac{p}{k})$ is the growth rate which is a linear decreasing function of p.

Least square interpolation of growth rate

$$R\,(t) = 0.01439454545 - 0.000045454545454545450182{*}t$$

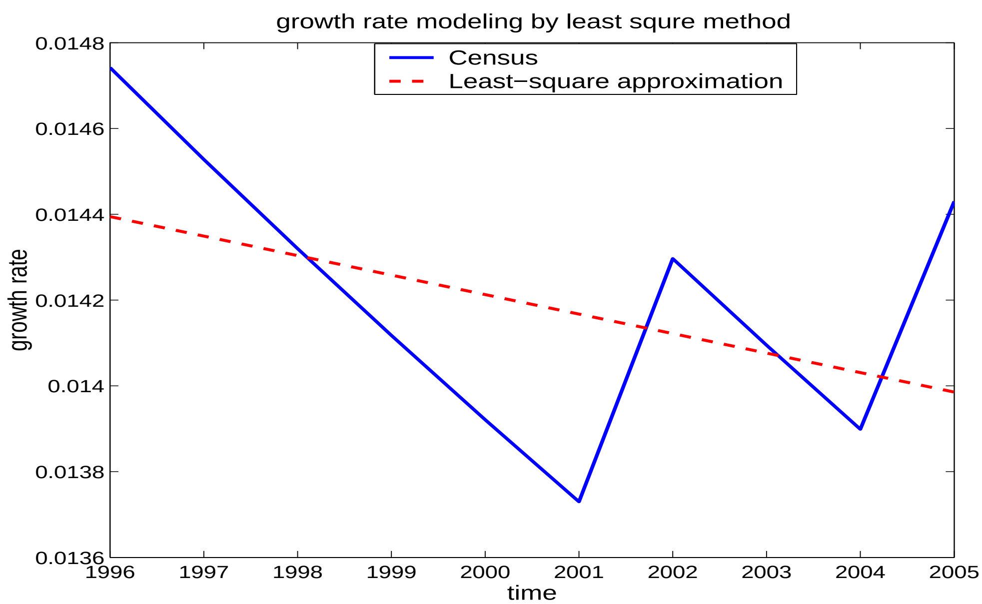


Figure shows the trend of the growth rate with respect to time. We approximate this trend by a linear profile which is shown by the exponential in the figure and our parameter 'm' is calculated as follows:

$$m = abs\left(\frac{r(4)-r(8)}{r(8)} - \frac{r(8)-r(12)}{r(12)}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.3)}$$

where r(12), r(8) and r(4) lie exact the on the line. Then we have used a Matlab program for calculating population of Bangladesh using (3.3) and taking a and k constant. We will see that the data from 1981 to 2035 is reasonable for the logistic model.

4.3. Future population projection of bangladesh by numerical solution of generalized logistic population model

Our logistic Model is

$$\frac{dp}{dt} = a(t)p(t)(1-\frac{p(t)}{k}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4)}$$

where k is assumed to be constant which is determined by the formula.

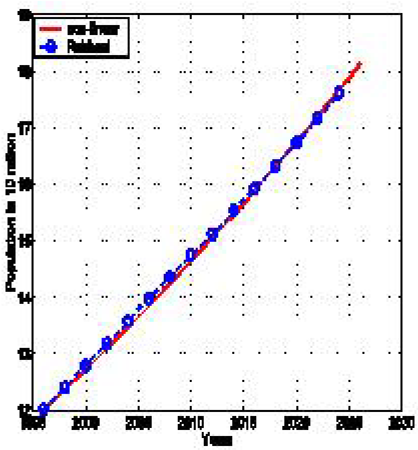
We determine a(t) as follows:

$$\frac{1}{p}\frac{dp}{dt} = a(t)(1-\frac{p(t)}{k}) = r(t), \text{ growth rate} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(3.5)}$$

So we have

$$a(t)=\frac{r(t)}{1-\frac{p(t)}{k}} \quad \text{………………………………………………..…..……(3.6)}$$

where $p(t)<k$ (carrying capacity).
To estimate $a(t)$, we model $r(t)$ with a linear equation in time
$r(t)=r(0)+mt$ ……………………………..…………………………………..(3.7)
where $m=(r(1996)-r(1991))/5$ is determined by the given data [10] and k is assumed to be constant is determined from this data using the formula which is approximately 190 million.



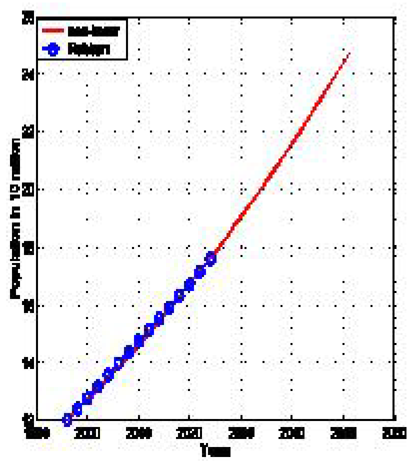


Table for the Projection population of Bangladesh:

Year	Census population	Population of non linear model	Year	Census population	Population of non linear model
1996	122.1	12.0200	2016		16.0953
1997	123.9	12.1939	2017		16.3124
1998	125.7	12.3697	2018		16.5316
1999	127.5	12.5475	2019		16.7530
2000	129.3	12.7272	2020		16.9766
2001	131.1	12.9090	2021		17.2024
2002	132.9	13.0927	2022		17.4304
2003	134.8	13.2784	2023		17.6606
2004	136.7	13.4662	2024		17.8931
2005	138.6	13.6559	2025		18.1277
2006	140.6	13.8478	2026		18.3646
2007		14.0416	2027		18.6037
2008		14.2376	2028		18.8451

2009		14.4355	2029		19.0887
2010		14.6356	2030		19.3345
2011		14.8378	2031		19.5827
2012		15.2484	2032		19.8331
2013		15.4569	2033		20.0857
2014		15.6676	2034		20.3407
2015		15.8804	2035		20.5979

5. Conclusion

In this study a mathematical analysis of the future population of Bangladesh is carried out based on an ordinary differential equation model which is called logistic model. In this model, at first, the vital coefficients a and b are assumed as constant. A formula is developed to calculate the vital coefficients when they are constant. However, considering growth rate as a non-linear equation we have also shown how to calculate a(t) as a function of t from the data and some MATLAB program are used for this purposes. MATLAB program based on an algorithm of Runge-Kutta scheme is developed for direct calculation of future population. First we test the numerical method for Bangladesh population data (1991-2006) and we find our population projection which is very good fit with the actual data. Then we establish a non-linear model that gives future population projection for Bangladesh at the time from 1996 to 2035. To make the non-linear model we use the least square interpolation of growth rate

$$R(t)=0.01439454545-0.000045454545454545450182*t$$

Here this result shows a very good argument with the results due to Rabbani [11] which has the predicted up to 2025.

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