On Semi Prime Ideals in Nearlattices

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Abstract. Recently Yehuda Rav has given the concept of Semi prime ideals in a general lattice by generalizing the notion of 0-distributive lattices. In this paper we study several properties of these ideals in a general nearlattice and include some of their characterizations. We give some results regarding maximal filters and include a number of Separation properties in a general nearlattice with respect to the annihilator ideals. We also include a Separation property for a filter disjoint to the semi prime ideal $\{x\}$. 

Keywords: 0-distributive nearlattice, prime ideal, semi-prime ideal, annihilator ideal, maximal filter

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1. Introduction

The concept of 0-distributive lattices was given by J.C.Varlet [6] in generalizing the concept of pseudocomplementation. In a bounded lattices $L$, for an element $a \in L$, $a^*$ is called the pseudocomplement of a if $a \land a^* = 0$ and for $x \in L$, $a \land x = 0$ implies $x \leq a^*$. In other words, the set of all elements disjoint to the element $a$ forms a principal ideal $(a^*)$. A lattice with 0 and 1 whose every element has a pseudocomplement, is called a pseudocomplemented lattice. By Varlet, a lattices $L$ with 0 is called 0-distributive if the set of all elements disjoint to element $a$ form an ideal (not necessarily principal ideal). Equivalently, $L$ with 0 is called 0-distributive if for all $a,b,c \in L$, $a \land b = 0 = a \land c$ imply $a \land (b \lor c) = 0$. Of course, every distributive lattice is 0-distributive. Also every pseudocomplemented
lattice is 0-distributive. Dually, we can study 1-distributive lattices if the lattices have 1.

It is easy to see that Pentagonal lattice (Figure 1) is 0-distributive but the Diamond lattice (Figure 2) is not.

For detailed literature on this topic, see [1] and [4].

Recently, Y. Rav [5] has generalized this concept and gave the definition of semi prime ideals in a lattice. An ideal I of a lattice L is called a semi prime ideal if for all \( x, y, z \in L \), \( x \land y \in I \) and \( x \land z \in I \) imply \( x \land (y \lor z) \in I \). Thus, for lattice L with 0, L is called 0-distributive if and only if \( (0) \) is a semi prime ideal. In a distributive lattice L, every ideal is a semi prime ideal. Moreover, every prime ideal is semi prime. In a pentagonal lattice (Figure 1) \( (0) \) is semi prime but not prime. Here \( (b) \) and \( (c) \) are prime, but \( (a) \) is not even semi prime. Again in Figure 2, \( (0) \), \( (a) \), \( (b) \), \( (c) \) are not semi prime.

In this paper we extend this concept for nearlattices and include a number of separation properties in a general nearlattice with respect to the annihilator ideals. Moreover, by studying a congruence related to Glivenko congruence we give a separation theorem related to separation properties in distributive nearlattices given by [4].

2. Semi Prime Ideals in a Nearlattice

A nearlattice \( S \) is a meet semilattice with the property that any two elements possessing a common upper bound, have a supremum. This property is known as the upper bound property. \( S \) is called a distributive nearlattice if for all \( x, y, z \in S \), \( x \land (y \lor z) = (x \land y) \lor (x \land z) \), provided \( y \lor z \) exists. Here right hand expression exists by the upper bound property. For detailed literature on nearlattices we refer the reader to consult [2] and [3]. By [7], a nearlattice \( S \) with 0 is called a 0-distributive nearlattice, if for all \( x, y, z \in S \), \( x \land y = 0 = x \land z \) imply \( x \land (y \lor z) = 0 \), provided \( y \lor z \) exists. Of course, every distributive nearlattice is 0-distributive. Since a nearlattice with 1 is a lattice (by the upper bound property), so we can not bring the idea of pseudocomplementation in a nearlattice. But [7] have proved that a nearlattices with 0 is 0-distributive if and only if the lattice of ideals I(S) is pseudocomplemented, which is also equivalent to I(S) is 0-distributive.
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For a non-empty subset $I$ of $S$, $I$ is called a down set if for $a \in I$ and $x \leq a$ imply $x \in I$. Moreover $I$ is an ideal if $a \lor b \in I$ for all $a, b \in S$, provided $a \lor b$ exists. Similarly, $F$ is called a filter of $S$ if for $a, b \in F$, $a \land b \in F$ and for $a \in F$ and $x \geq a$ imply $x \in F$. $F$ is called a maximal filter if for any filter $M \supseteq F$ implies either $M = F$ or $M = L$. A proper ideal(down set) $I$ is called a prime ideal(down set) if for $a, b \in S, a \land b \in I$ imply either $a \in I$ or $b \in I$. A prime ideal $P$ is called a minimal prime ideal if it does not contain any other prime ideal. Similarly, a proper filter $Q$ is called a prime filter if $a \lor b \in Q$ $(a, b \in S)$ when $a \lor b$ exists, implies either $a \in Q$ or $b \in Q$. It is very easy to check that $F$ is a filter of $S$ if and only if $S - F$ is a prime down set. Moreover, $F$ is a prime filter if and only if $S - F$ is a prime ideal.

An ideal $I$ of a nearlattice $S$ is called a semi prime ideal if for all $x, y, z \in L$, $x \land y \in I$ and $x \land z \in I$ imply $x \land (y \lor z) \in I$ provided $y \lor z$ exists. Thus, for nearlattice $S$ with 0, $S$ is called 0-distributive if and only if $[0]$ is a semi prime ideal. In a distributive nearlattice $S$, every ideal is a semi prime ideal. Moreover, every prime ideal is semi prime. In the nearlattice of figure 3,

![Figure 3](image1.png)

(b) and (d) are prime, (c) is not prime but semi prime and (a) is not even semi prime. Again in figure 4, (0), (a), (b), (c) and (d) are not semi prime.

**Lemma 1.** Non empty intersection of all prime (semi prime) ideals of a nearlattice is a semi-prime ideal.

**Proof.** Let $a, b, c \in S$ and $I = \bigcap\{P : P$ is a prime ideal $\}$ and $I$ is nonempty. Let $a \land b \in I$ and $a \land c \in I$. Then $a \land b \in P$ and $a \land c \in P$ for all $P$. Since each $P$ is prime (semi prime), so $a \land (b \lor c) \in P$ for all $P$. Hence $a \land (b \lor c) \in I$, and so $I$ is semi-prime. 

**Corollary 2.** Intersection of two prime(semi prime) ideals is a semi-prime ideal. 

**Lemma 3.** Every filter disjoint from an ideal $I$ is contained in a maximal filter disjoint from $I$. 

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Proof. Let $F$ be a filter in $L$ disjoint from $I$. Let $\mathcal{F}$ be the set of all filters containing $F$ and disjoint from $I$. Then $\mathcal{F}$ is nonempty as $F \in \mathcal{F}$. Let $C$ be a chain in $\mathcal{F}$ and let $M = \bigcup (X : X \in C)$. We claim that $M$ is a filter. Let $x \in M$ and $y \geq x$. Then $x \in X$ for some $X \in C$. Hence $y \in X$ as $X$ is a filter. Therefore, $y \in M$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since $C$ is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$. Then $x \wedge y \in Y$ and so $x \wedge y \in M$. Moreover, $M \supseteq F$. So $M$ is a maximum element of $C$. Then by Zorn’s Lemma, $\mathcal{F}$ has a maximal element, say $Q \supseteq F$. ●

Lemma 4. Let $I$ be an ideal of a nearlattice $S$. A filter $M$ disjoint from $I$ is a maximal filter disjoint from $I$ if and only if for all $a \notin M$, there exists $b \in M$ such that $a \wedge b \in I$.

Proof. Let $M$ be maximal and disjoint from $I$ and $a \notin M$. Let $a \wedge b \notin I$ for $b \in M$. Consider $M_1 = \{ y \in L : y \geq a \wedge b, \ b \in M \}$. Clearly $M_1$ is a filter. For any $b \in M$, $b \geq a \wedge b$ implies $b \in M_1$. So $M_1 \supseteq M$. Also $M_1 \cap I = \emptyset$. For if not, let $x \in M_1 \cap I$. This implies $x \in I$ and $x \geq a \wedge b$ for some $b \in M$. Hence $a \wedge b \in I$, which is a contradiction. Hence $M_1 \cap I \neq \emptyset$. Now $M \subsetneq M_1$ because $a \notin M$ but $a \in M_1$. This contradicts the maximality of $M$. Hence there exists $b \in M$ such that $a \wedge b \in I$.

Conversely, if $M$ is not maximal disjoint from $I$, then there exists a filter $N \supseteq M$ and disjoint with $I$. For any $a \in N - M$, there exists $b \in M$ such that $a \wedge b \in I$. Hence, $a, b \in N$ implies $a \wedge b \in I \cap N$, which is a contradiction. Hence $M$ must be a maximal filter disjoint with $I$. ●

Let $S$ be a nearlattice with 0. For $A \subseteq S$, we define $A^\perp = \{ x \in L : x \wedge a = 0 \text{ for all } a \in A \}$. $A^\perp$ is always down set of $S$. Moreover, it is convex but it is not necessarily an ideal.

Theorem 5. Let $S$ be a 0-distributive nearlattice. Then for $A \subseteq S$, $A^\perp = \{ x \in L : x \wedge a = 0 \text{ for all } a \in A \}$ is a semi-prime ideal.

Proof. We have already mentioned that $A^\perp$ is a down set of $S$. Let $x, y \in A^\perp$ and $x \vee y$ exists. Then $x \wedge a = 0 = y \wedge a$ for all $a \in L$. Since $S$ is 0-distributive, so $a \wedge (x \vee y) = 0$ for all $a \in A$. This implies $x \vee y \in A^\perp$ and so $A^\perp$ is an ideal.

Now let $x \wedge y \in A^\perp$ and $x \wedge z \in A^\perp$ and $y \vee z$ exists. Then $x \wedge y \wedge a = 0 = x \wedge z \wedge a$ for all $a \in A$. This implies $(x \wedge a) \wedge y = 0 = (x \wedge a) \wedge z$ and so by 0-distributivity again, $x \wedge a \wedge (y \vee z) = 0$ for all $a \in L$. Hence $x \wedge (y \vee z) \in A^\perp$ and so $A^\perp$ is a semi prime ideal. ●
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Let $A \subseteq S$ and $J$ be an ideal of $S$. We define

$$A^{\perp J} = \{ x \in L : x \land a \in J \ \text{for all} \ a \in A \}.$$ 

This is clearly a down set containing $J$. In presence of distributivity, this is an ideal. $A^{\perp J}$ is called an annihilator of $A$ relative to $J$. We denote $I_J(S)$, by the set of all ideals containing $J$. Of course, $I_J(S)$ is a bounded lattice with $J$ and $S$ as the smallest and the largest elements. If $A \in I_J(S)$, and $A^{\perp J}$ is an ideal, then $A^{\perp J}$ is called an annihilator ideal and it is the pseudo complement of $A$ in $I_J(S)$.

**Theorem 6.** Let $A$ be a non-empty subset of a nearlattice $S$ and $J$ be an ideal of $S$. Then

$$A^{\perp J} = \bigcap (P : P \text{ is minimal prime down set containing } J \text{ but not containing } A) .$$

**Proof.** Suppose $X = \bigcap (P : A \not\subset P, P \text{ is a minimal prime down set})$. Let $x \in A^{\perp J}$. Then $x \land a \in J$ for all $a \in A$. Choose any $P$ of right hand expression. Since $A \not\subset P$, there exists $z \in A$ but $z \not\in P$. Then $x \land z \in J \subseteq P$. So $x \in P$, as $P$ is prime. Hence $x \in X$.

Conversely, let $x \in X$. If $x \not\in A^{\perp J}$, then $x \land b \not\in J$ for some $b \in A$. Let $D = [x \land b]$. Hence $D$ is a filter disjoint from $J$. Then by Lemma 3, there is a maximal filter $M \supseteq D$ but disjoint from $J$. Then $S - M$ is a minimal prime down set containing $J$. Now $x \not\in S - M$ as $x \in D$ implies $x \in M$. Moreover, $A \not\subseteq S - M$ as $b \in A$, but $b \in M$ implies $b \not\in S - M$, which is a contradiction to $x \in X$. Hence $x \in A^{\perp J}$. 

Following Theorem gives some nice characterizations semi prime ideals.

**Theorem 7.** Let $S$ be a nearlattice and $J$ be an ideal of $S$. The following conditions are equivalent.

1. $J$ is semi prime.
2. $\{ a \}^{\perp J} = \{ x \in L : x \land a \in J \}$ is a semi prime ideal containing $J$.
3. $A^{\perp J} = \{ x \in L : x \land a \in J \ \text{for all} \ a \in A \}$ is a semi prime ideal containing $J$.
4. $I_J(S)$ is pseudo complemented.
5. $I_J(S)$ is a $0$-distributive lattice.
6. Every maximal filter disjoint from $J$ is prime.

**Proof.** (i)$\Rightarrow$(ii). $\{ a \}^{\perp J}$ is clearly a down set containing $J$. Now let $x, y \in \{ a \}^{\perp J}$ and $x \lor y$ exists. Then $x \land a \in J, y \land a \in J$. Since $J$ is semi prime, so $a \land (x \lor y) \in J$. This implies $x \lor y \in \{ a \}^{\perp J}$, and so it is an ideal containing $J$.

Now let $x \land y \in \{ a \}^{\perp J}$ and $x \land z \in \{ a \}^{\perp J}$ with $y \lor z$ exists. Then $x \land y \land a \in J$.
and \( x \land z \land a \in J \). Thus, \( (x \land a) \land y \in J \) and \( (x \land a) \land z \in J \). Then \( (x \land a) \land (y \lor z) \in J \), as \( J \) is semi prime. This implies \( x \land (y \lor z) \in \{a\}^{1/2} \), and so \( \{a\}^{1/2} \) is semi prime.

(ii) \( \Rightarrow \) (iii). This is trivial by Lemma 1, as \( A^{1/2} = \cap \{a\}^{1/2} ; a \in A \} \).

(iii) \( \Rightarrow \) (iv). Since for any \( A \in I_j(S) \), \( A^{1/2} \) is an ideal, it is the pseudo complement of \( A \) in \( I_j(S) \), so \( I_j(S) \) is pseudo complemented.

(iv) \( \Rightarrow \) (v). This is trivial as every pseudo complemented lattice is 0-distributive.

(v) \( \Rightarrow \) (vi). Let \( I_j(S) \) is 0-distributive. Suppose \( F \) is a maximal filter disjoint from \( J \). Suppose \( f, g \notin F \) and \( f \lor g \) exists. By Lemma 5, there exist \( a, b \in F \) such that \( a \land f \in J, b \land g \in J \). Then \( f \land a \land b \in J \), \( g \land a \land b \in J \). Hence \( (f) \land (a \land b) \subseteq J \) and \( (g) \land (a \land b) \subseteq J \). Then \( (f \lor g) \land (a \land b) \subseteq J \), by the 0-distributive property of \( I_j(S) \). Hence, \( (f \lor g) \land a \land b \in J \). This implies \( f \lor g \notin F \) as \( F \cap J = \emptyset \), and so \( F \) is prime.

(vi) \( \Rightarrow \) (i) Let (vi) holds. Suppose \( a, b, c \in S \) with \( a \land b \in J, a \land c \in J \) with \( b \lor c \) exists. If \( a \land (b \lor c) \notin J \), then \( [a \land (b \lor c)] \cap J = \emptyset \). Then by Lemma 3, there exists a maximal filter \( F \supseteq [a \land (b \lor c)] \) and disjoint from \( J \). Then \( a \in F, b \lor c \in F \). By (vi) \( F \) is prime. Hence either \( a \land b \in F \) or \( a \land c \in F \). In any case \( J \cap F \neq \emptyset \), which gives a contradiction. Hence \( a \land (b \lor c) \in J \), and so \( J \) is semi prime.

Corollary 8. In a nearlattice \( S \), every filter disjoint to a semi-prime ideal \( J \) is contained in a prime filter.

Proof. This immediately follows from Lemma 3 and Theorem 7. 

Theorem 9. If \( J \) is a semi-prime ideal of a nearlattice \( S \) and \( J \neq A = \bigcap \{J \_ \land a \} ; J \_ \land a \) is an ideal containing \( J \)_, Then \( A^{1/2} = \{x \in L ; \{x\}^{1/2} \neq J \} \).

Proof. Let \( x \in A^{1/2} \). Then \( x \land a \in J \) for all \( a \in A \). So \( a \in \{x\}^{1/2} \) for all \( a \in A \). Then \( A \subseteq \{x\}^{1/2} \) and so \( \{x\}^{1/2} \neq J \). Conversely, let \( x \in S \) such that \( \{x\}^{1/2} \neq J \). Since \( J \) is semi-prime, so \( \{x\}^{1/2} \) is an ideal containing \( J \). Then \( A \subseteq \{x\}^{1/2} \), and so \( A^{1/2} \supseteq \{x\}^{1/2 \lor 1/2} \). This implies \( x \in A^{1/2} \), which completes the proof. 


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In [1], the authors have provided a series of characterizations of 0-distributive lattices. Here we give some results on semi prime ideals related to their results for nearlattices.

**Theorem 10.** Let $S$ be a nearlattice and $J$ be an ideal. Then the following conditions are equivalent.

(i) $J$ is semi-prime.

(ii) Every maximal filter of $S$ disjoint with $J$ is prime.

(iii) Every minimal prime down set containing $J$ is a minimal prime ideal containing $J$.

(iv) Every filter disjoint with $J$ is disjoint from a minimal prime ideal containing $J$.

(v) For each element $a \notin J$, there is a minimal prime ideal containing $J$ but not containing $a$.

(vi) Each $a \notin J$ is contained in a prime filter disjoint to $J$.

**Proof.** (i) $\iff$ (ii) follows from Theorem 7.

(ii) $\implies$ (iii). Let $A$ be a minimal prime down set containing $J$. Then $S-A$ is a maximal filter disjoint with $J$. Then by (ii) $S-A$ is prime and so $A$ is a minimal prime ideal.

(iii) $\implies$ (ii). Let $F$ be a maximal filter disjoint with $J$. Then $S-F$ is a minimal prime down set containing $J$. Thus by (iii), $S-F$ is a minimal prime ideal and so $F$ is a prime filter.

(i) $\implies$ (iv). Let $F$ a filter of $S$ disjoint from $J$. Then by Corollary 8, there is a prime filter $Q \supseteq F$ and disjoint from $F$.

(iv) $\implies$ (v). Let $a \in S$, $a \notin J$. Then $[a] \cap J = \emptyset$. Then by (iv) there exists a minimal prime ideal $A$ disjoint from $[a]$. Thus $a \notin A$.

(v) $\implies$ (vi). Let $a \in L$, $a \notin J$. Then by (v) there exists a minimal prime ideal $P$ such that $a \notin P$. Implies $a \in S-P$ and $S-P$ is a prime filter.

(vi) $\implies$ (i). Suppose $J$ is not semi-prime. Then there exists $a, b, c \in L$ such that $a \land b \in J$, $a \land c \in J$ and $b \lor c$ exists, but $a \land (b \lor c) \notin J$. Then by (vi) there exists a prime filter $Q$ disjoint from $J$ and $a \land (b \lor c) \in Q$. Let $F = [a \land (b \lor c)]$. Then $J \cap F = \emptyset$ and $F \subseteq Q$. Now $a \land (b \lor c) \in Q$ implies $a \in Q$, $b \lor c \in Q$. Since $Q$ is prime so either $a \land b \in Q$ or $a \land c \in Q$. This gives a contradiction to the fact that $Q \cap J = \emptyset$. Therefore, $a \land (b \lor c) \in J$ and so $J$ is semi-prime. 

Now we give another characterization of semi-prime ideals with the help of Prime Separation Theorem using annihilator ideals.

**Theorem 11.** Let $J$ be an ideal in a nearlattice $S$. $J$ is semi-prime if and only if for all filter $F$ disjoint to $\{x\}^\bot$, there is a prime filter containing $F$ disjoint to $\{x\}^\bot$. 

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Proof. Using Zorn’s Lemma we can easily find a maximal filter \( Q \) containing \( F \) and disjoint to \( \{x_j^{\perp}\}_j \). We claim that \( x \in Q \). If not, then \( Q \cup [x] \supseteq Q \). By maximality of \( Q \), \( (Q \cup [x]) \cap \{x_j^{\perp}\}_j \neq \emptyset \). If \( t \in (Q \cup [x]) \cap \{x_j^{\perp}\}_j \), then \( t \geq q \wedge x \) for some \( q \in Q \) and \( t \wedge x \in J \). This implies \( q \wedge x \in J \) and so \( q \in \{x_j^{\perp}\}_j \) gives a contradiction. Hence \( x \in Q \).

Now, let \( z \notin Q \). Then \( (Q \cup [z]) \cap \{x_j^{\perp}\}_j \neq \emptyset \). Suppose \( y \in (Q \cup [z]) \cap \{x_j^{\perp}\}_j \) then \( y \geq q \wedge z \) and \( y \wedge z \in J \) for some \( q_j \in Q \). This implies \( q \wedge x \wedge z \in J \) and \( q \wedge x \in Q \). Hence by Lemma 4, \( Q \) is a maximal filter disjoint to \( \{x_j^{\perp}\}_j \). Then by Theorem 7, \( Q \) is prime.

Conversely, let \( x \wedge y \in J \), \( x \wedge z \in J \) and \( y \wedge z \) exists. If \( x \wedge (y \wedge z) \notin J \), then \( y \wedge z \notin \{x_j^{\perp}\}_j \). Thus \( [y \wedge z] \cap \{x_j^{\perp}\}_j = \emptyset \). So there exists a prime filter \( Q \) containing \( [y \wedge z] \) and disjoint from \( \{x_j^{\perp}\}_j \). As \( y, z \in \{x_j^{\perp}\}_j \), so \( y, z \notin Q \). Thus \( y \wedge z \notin Q \), as \( Q \) is prime. This implies \( [y \wedge z] \not\subseteq Q \), a contradiction. Hence \( x \wedge (y \wedge z) \in J \), and so \( J \) is semi-prime.

Here is another characterization of semi-prime ideals.

**Theorem 12.** Let \( J \) be a semi-prime ideal of a nearlattice \( S \) and \( x \in S \). Then a prime ideal \( P \) containing \( \{x_j^{\perp}\}_j \) is a minimal prime ideal containing \( \{x_j^{\perp}\}_j \) if and only if for \( p \in P \), there exists \( q \in S - P \) such that \( p \wedge q \in \{x_j^{\perp}\}_j \).

**Proof.** Let \( P \) be a prime ideal containing \( \{x_j^{\perp}\}_j \) such that the given condition holds. Let \( K \) be a prime ideal containing \( \{x_j^{\perp}\}_j \) such that \( K \subseteq P \). Let \( p \in P \). Then there is \( q \in S - P \) such that \( p \wedge q \in \{x_j^{\perp}\}_j \). Hence \( p \wedge q \in K \). Since \( K \) is prime and \( q \notin K \), so \( p \in K \). Thus, \( P \subseteq K \) and so \( K = P \). Therefore, \( P \) must be a minimal prime ideal containing \( \{x_j^{\perp}\}_j \).

Conversely, let \( P \) be a minimal prime ideal containing \( \{x_j^{\perp}\}_j \). Let \( p \in P \). Suppose for all \( q \in S - P \), \( p \wedge q \notin \{x_j^{\perp}\}_j \). Let \( D = (S - P) \cup \{p\} \). We claim that \( \{x_j^{\perp}\}_j \cap D = \emptyset \). If not, let \( y \in \{x_j^{\perp}\}_j \cap D \). Then \( p \wedge q \leq y \in \{x_j^{\perp}\}_j \), which is a contradiction to the assumption. Then by Theorem 11, there exists a maximal (prime) filter \( Q \supseteq D \) and disjoint to \( \{x_j^{\perp}\}_j \). By the proof of Theorem 11, \( x \in Q \). Let \( M = S - Q \). Then \( M \) is a prime ideal. Since \( x \in Q \), so \( t \wedge x \in J \subseteq M \) implies \( t \in M \) as \( M \) is prime. Thus \( \{x_j^{\perp}\}_j \subseteq M \). Now \( M \cap D = \emptyset \). This implies \( M \cap (S - P) = \emptyset \) and hence \( M \subseteq P \). Also \( M \neq P \), because \( p \in D \) implies \( p \notin M \) but \( p \in P \). Hence \( M \) is a prime ideal.
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containing \( \{x\} \) which is properly contained in \( P \). This gives a contradiction to the minimal property of \( P \). Therefore the given condition holds.

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