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An Effective Modification to Solve Transportation Problems: A Cost Minimization Approach

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Abstract. It is well-known that Linear Programming Problem (LPP) is one of the most potential mathematical tools for efficient allocation of operational resources. Many problems in real situation can be formulated as LPP. When a situation can be entirely modeled as a network, very efficient algorithms exist for the solution of the optimization problem which is many times more efficient than the solution methods of LPP. Transportation problems (TP), as is known, are a basic network problem which can be formulated as a LPP. The main objective of TP is to minimize the transportation cost of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. ware houses). It is to be mentioned that Balanced TP and Unbalanced TP are the types of TP. If the sum of the supplies of all the sources is equal to the sum of the demands of all the destinations, the problem is termed as a balanced transportation problem. Again, if the sum of the supplies of all the sources is not equal to the sum of the demand of all the destinations, the problem is termed as unbalanced transportation problem. Here we have developed a new method of finding an Initial Basic Feasible Solution (IBFS) for both the Balanced TP and Unbalanced TP.

Keywords: LPP, TP, Transportation Cost, IBFS

AMS Mathematical Subject Classification (2010): 05C78

1. Introduction

The first algorithm in solving the TP was developed by G. B. Dantzig [6] and referred as North West Corner Method (NWCM) by Charnes and Cooper. This is the method of finding an IBFS of TP which consider the north-west-corner cost cell at every stage of allocation. Then the Least Cost Method (LCM) [2,9] consists in allocating as much as possible in the lowest cost cell of the Transportation Table (TT) in making allocation in every stage. Vogel's Approximation Method (VAM) [7,10,11,14] and Extremum Difference Method (EDM) [8] provides comparatively better Initial Basic Feasible Solution. The problem of minimizing transportation cost has been studied since long and is well known [7,8,11,13,14]. In this work we have added a new algorithm that provides a better IBFS, for both the balanced and unbalanced TP, than those algorithms just mentioned.

TP in general are concerned with distributing any single commodity from any group of supply centre, called sources, to any group of receiving centre, called destinations. A destination can receive its demand from one or more sources. Each source has a fixed supply of units, where the entire supply must be distributed to the destinations. Similarly, each destination has fixed demand of units, where the entire demand must be received from the sources.

2. Cost minimization TP

A cost minimization TP is formulated as

$$\text{Minimize:} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le a_i \quad ; \quad i = 1, 2, \dots \dots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad ; \quad j = 1, 2, \dots \dots, n$$

$$x_{ij} \ge 0 \quad \text{for all } i \text{ and } j.$$

where

i =1, 2,……, m is the set of origins.

j =1, 2,……, n is the set of destinations.

x_{ij} = the quantity transported from the i-th origin to the j-th destination.

c_{ij} = per unit cost in transporting goods from i-th origin to the j-th destination.

a_i = the amount available at the i-th origin.

b_j = the demand of the j-th destination.

3. Proposed method of finding IBFS

Idea of this method is developed from VAM. The proposed method can be applied to solve all types of TP. Procedure of finding an IBFS using this method is illustrated below.

Step 1 Subtract each of the elements of every row from the largest entry of that row of the Transportation Table and place them on the right-top of the corresponding elements.

Step 2 Apply the same operation on each of the elements of every column and place them on the right-bottom of the corresponding elements.

Step 3 Form a modified transportation table whose elements remain same and place the magnitude of the subtraction of right-top and right-bottom entry of Step 1 and Step 2 on the right-bottom of the corresponding elements.

Step 4 Place the row distribution indicators (RDI) and the column distribution indicators (CDI) just after and below the supply limits and demand requirements respectively within first bracket, which are the difference of the largest and nearest-to-largest right bottom entries. If there are two or more largest entries, the result is to be considered as zero.

Step 5 Identify the highest distribution indicator. Choose the lowest element along the highest distribution indicator. If there are two or more highest indicators, choose the highest indicator along which the smallest cost unit is present. If there are two or more number of smallest elements, choose any one of them arbitrarily.

Step 6 Allocate x_{ij} = min(a_i, b_j) on the left top of the smallest entry in the (i, j)-th cell (TT of Step-3).

Step 7 If $a_i < b_j$, leave the *i*-th row and readjust b_j as $b_j^{/} = b_j - a_i$.
If $a_i > b_j$, leave the *j*-th column and readjust a_i as $a_i^{/} = a_i - b_j$.
If $a_i = b_j$, leave either *i*-th row or *j*-th column but not both.

Step 8 Repeat Steps 4 to 7 until the rim requirements are satisfied.

Step 9 Calculate $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$, z being the minimum transportation cost.

For an unbalanced TP we shall solve the problem by following the above algorithm without doing any operation on dummy destination/supply elements. During the operation of Step 4, if we find a single entry, that entry will be considered as row or column distribution indicator. Finally we shall allocate the demand/supply of the dummy elements by following simple arithmetic calculation.

4. Numerical illustrations

4.1. Example 1(Balanced TP): A company manufactures motor tyres and it has four factories F_1, F_2, F_3 and F_4 whose weekly production capacities are 5, 8, 7 and 14 thousand pieces of tyres respectively. The company supplies tyres to its three showrooms located at D_1, D_2 and D_3 whose weekly demand are 7, 9 and 18 thousand pieces respectively. The transportation cost per thousand pieces of tyre is given below in the TT:

		Showrooms			
		D_1	D_2	D_3	Supply
Factories	F_1	2	7	4	5
	F_2	3	3	1	8
	F_3	5	4	7	7
	F_4	1	6	2	14
Demand		7	9	18	

Table 4.1.1. An example of balanced TP

We want to schedule the shifting of tyres from factories to showrooms with a minimum cost.

Since the factories demand 34 units equals the total supply 34 units, the given problem is a balanced TP.

Applying the algorithm of the **Proposed Method**; the row differences and the column differences are shown on the right top and right bottom respectively to each of the elements.

Factories	Showrooms			Supply
	D_1	D_2	D_3	
F_1	2^5_3	7^0_0	4^3_3	5
F_2	3^0_2	3^0_4	1^2_6	8
F_3	5^2_0	4^3_3	7^0_0	7
F_4	1^5_4	6^0_1	2^4_5	14
Demand	7	9	18	

Table 4.1.2. Modified TT

The distribution is made according to proposed algorithm is

Factories	Showrooms			Supply	RDI		
	D_1	D_2	D_3				
F_1	$^{5}2_2$	7_0	4_0	5	(2)	(2)	--
F_2	3_2	3_4	$^{8}1_4$	8	(0)	--	--
F_3	5_2	$^{7}4_0$	7_0	7	(2)	(2)	(2)
F_4	$^{2}1_1$	$^{2}6_1$	$^{10}2_1$	14	(0)	(0)	(0)
Demand	7	9	18				
CDI	(0) (0) (1)	(3) (1) (1)	(3) (1) (1)				

Table 4.1.3. Initial solution tableau of proposed method

Therefore, the solution for the given problem is

$$x_{11} = 5,\ x_{23} = 8,\ x_{32} = 7,\ x_{41} = 2,\ x_{42} = 2 \text{ and } x_{43} = 10 .$$

and the total transportation cost is

$$z = 2\times5 + 1\times8 + 4\times7 + 1\times2 + 6\times2 + 2\times10$$
$$= 10 + 8 + 28 + 2 + 12 + 20$$
$$= 80$$

4.2. Example 2 (Unbalanced TP): A company has four plants at locations A, B, C and D, which supply to warehouses located at E, F, G, H and I. Monthly plant capacities are 300, 500, 825 and 375 units respectively. Monthly warehouse requirements are 350, 400, 250, 150 and 400 units respectively. Unit transportation costs are given below.

	Warehouses					
Plants	E	F	G	H	I	Capacities
A	10	2	16	14	10	300
B	6	18	12	13	16	500
C	8	4	14	12	10	825
D	14	22	20	8	18	375
Requirements	350	400	250	150	400	

Table 4.2.1. An example of unbalanced TP

Determine a distribution plan for the company in order to minimize the total transportation cost.

Since the warehouse requirements 1550 units is less than the total plant capacities 2000 units, the given problem is an unbalanced TP. We introduce a dummy warehouse W having all the transportation costs equal to zero to make the TP balanced.

Applying the algorithm of the Proposed Method; the row differences and the column differences are shown on the right top and right bottom respectively to each of the elements excluding the dummy one.

Plants	Warehouses						Capacities
	E	F	G	H	I	W	
A	10^{6}_{4}	2^{14}_{20}	16^{0}_{4}	14^{2}_{0}	10^{6}_{8}	0	300
B	6^{12}_{8}	18^{0}_{4}	12^{6}_{8}	13^{5}_{1}	16^{2}_{2}	0	500
C	8^{6}_{6}	4^{10}_{18}	14^{0}_{6}	12^{2}_{2}	10^{4}_{8}	0	825
D	14^{8}_{0}	22^{0}_{0}	20^{2}_{0}	8^{14}_{6}	18^{4}_{0}	0	375
Requirements	350	400	250	150	400	450	

Table 4.2.2. Modified TT

According to our algorithm, the initial distribution is made without doing any operation on dummy destination/supply elements is

Plants	Warehouses						Capacities	RDI
	E	F	G	H	I	W		
A	10_{2}	$^{300}2_{6}$	16_{4}	14_{2}	10_{2}	0	300	(2) (2) (2) - - -
B	$^{350}6_{4}$	18_{4}	$^{150}12_{2}$	13_{4}	16_{2}	0	500	(0) (0) (2) (2) (2) -
C	8_{0}	$^{100}4_{8}$	$^{100}14_{6}$	12_{0}	$^{400}10_{4}$	0	825	(2) (2) (2) (2) (2) (2)
D	14_{8}	22_{0}	20_{2}	$^{150}8_{8}$	18_{4}	0	375	(0) (4) (2) (2) (2) (2)
Requirements	350	400	250	150	400	450		
CDI	(4)	(2)	(2)	(4)	(0)	-		
	-	(2)	(2)	(4)	(0)	-		
	-	(2)	(2)	-	(0)	-		
	-	(4)	(4)	-	(0)	-		
	-	-	(4)	-	(0)	-		
	-	-	(4)	-	(0)	-		

Table 4.2.3. Initial distribution for unbalanced TP

Now, the final allocation is,

Plants	Warehouses						Capacities
	E	F	G	H	I	W	
A	10	300 2	16	14	10	0	300
B	350 6	18	150 12	13	16	0	500
C	8	100 4	100 14	12	400 10	225 0	825
D	14	22	20	150 8	18	225 0	375
Requirements	350	400	250	150	400	450	

Table 4.2.4. Initial solution tableau of proposed method

Therefore, the solution for the given problem is

$$x_{12} = 300,\ x_{21} = 350,\ x_{23} = 150,\ x_{32} = 100\ ,\ x_{33} = 100,$$
$$x_{35} = 400,\ x_{36} = 225,\ x_{44} = 150 \quad \text{and} \quad x_{46} = 225.$$

and total transportation cost is

$$z = 2\times300 + 6\times350 + 12\times150 + 4\times100 + 14\times100 + 10\times400 + 0\times225 + 8\times150 + 0\times225$$
$$= 600 + 2100 + 1800 + 400 + 1400 + 4000 + 0 + 1200 + 0$$
$$= 11500.$$

5. Comparison of Transportation Cost Obtained by Different Methods

A comparative study among the results obtained by proposed method, existing methods and optimal solution is also carried out in the table 5.1.1.

Method	Example 1	Example 2
NWCM	102	19700
LCM	83	13100
VAM	80	12250
EDM	83	12250
Proposed Method	80	11500
Optimal Solution	76	11500

Table 5.1.1. Comparative study of the Results

5. Conclusion

In today's highly competitive market the pressure is increasing rapidly to the organizations to determine the better ways to deliver goods to the customers. That is why different organizations want to deliver products to the customers in a cost effective way and thus market becomes competitive. For this, Transportation model provides a powerful framework to meet this challenge.

The proposed method of finding an IBFS for the minimization of transportation cost is illustrated numerically. It is observed that proposed algorithm provides comparatively a better IBFS solution than those obtained by the traditional algorithms which is either optimal or near to optimal solution.

We would finally conclude that our developed algorithm provides a remarkable IBFS by ensuring minimum transportation cost which may be an attractive alternative to the traditional transportation problem solution methods.

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