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Connective Eccentricity Index of Some Thorny Graphs

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Abstract. The connective eccentricity index of a simple connected graph G is defined as $C^{\xi}(G)=\sum_{v\in V(G)}\frac{d_G(v)}{\varepsilon_G(v)}$, where $\varepsilon_G(v)$ and $d_G(v)$ respectively denote the eccentricity and the degree of the vertex v in G. The thorny graphs of G are obtained by attaching a number of thorns i.e., degree one vertices to each vertex of G. In this paper, we derive explicit expressions for the connective eccentricity index of some classes of thorny graphs.

***Keywords*:** Eccentricity; graph invariant; connective eccentric index; thorn graph; Corona product.

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1. Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. Also let m and n denote the number of vertices and edges of the graph. The connective eccentricity index of a graph G was introduced by Gupta, Singh and Madan [1] and was defined as $C^{\xi}(G)=\sum_{v\in V(G)}\frac{d_G(v)}{\varepsilon_G(v)}$, where $\varepsilon_G(v)$ and $d_G(v)$ respectively denote the eccentricity and the degree of the vertex v in G.

In [2], Ghorbani computed some bounds of connective eccentricity index and explicit expression for this index for two infinite classes of dendrimers. One of the present authors, in [3], presented some bounds for this connective eccentric index in terms of different graph invariants. Ghorbani and Malekjani in [4], compute the eccentric connectivity index and the connective eccentric index of an infinite family of fullerenes. Yu and Feng in [5], derived some upper or lower bounds for the connective eccentric index and found the maximal and the minimal values of connective eccentricity index among all n-vertex graphs with fixed number of pendent vertices. The present authors have also studied that index on some graph operations [6].

Let G be a given graph with vertex set $\{v_1, v_2, ..., v_n\}$ and $\{p_1, p_2, ..., p_n\}$ be a set of non-negative integers. Then, the thorn graph of G denoted by $G^*(p_1, p_2, ..., p_n)$ is obtained by attaching p_i pendant vertices to v_i for each i. This notion of thorn graph was introduced by Gutman in [7] and a number of study on thorn graphs for different topological indices are made by several researchers in the recent past. Very recently, De [8, 9] studied different eccentricity related topological indices on thorn graphs. In this paper, we derive explicit expressions for the connective eccentricity index of some classes of thorny graphs.

2. The Thorny complete graph

Let K_n be the compete graph with n vertices. The thorny graph K_n^* is obtained from K_n by attaching p_i thorns at every vertex of K_n, $i = 1,2,\ldots,n$. Let T be the total number of thorns attached to K_n.

Theorem 2.1. The connective eccentricity index of K_n^* is given by

$$C^{\xi}(K_n^*) = |E(K_n)| + \frac{5}{6}T .$$

Proof: Let the vertices of K_n are denoted by v_i, $i = 1, 2, ..., n$, and the newly attached pendent vertices are denoted by v_{ij}, $i = 1, 2, ..., n;\ j = 1, 2, ..., p_i$. Therefore, the degree and eccentricity of the vertices of K_n^* are given by $d_{K_n^*}(v_i) = n - 1 + p_i$, $d_{K_n^*}(v_{ij}) = 1$, $\varepsilon_{K_n^*}(v_i) = 2$, $\varepsilon_{K_n^*}(v_{ij}) = 3$, for $i = 1, 2, ..., n;\ j = 1, 2, ..., p_i$. Thus the connective eccentricity index of K_n^* is given by

$$C^{\xi}(K_n^*) = \sum_{i=1}^{n} \frac{d_{K_n^*}(v_i)}{\varepsilon_{K_n^*}(v_i)} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{d_{K_n^*}(v_{ij})}{\varepsilon_{K_n^*}(v_{ij})}$$

$$= \sum_{i=1}^{n} \frac{n - 1 + p_i}{2} + \sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{1}{3}$$

$$= \frac{n(n-1)}{2} + \frac{1}{2} \sum_{i=1}^{n} p_i + \frac{1}{3} \sum_{i=1}^{n} p_i = \frac{n(n-1)}{2} + \frac{5}{6}T$$

from where the desired result follows. □

3. The Thorny complete bipartite graph

Let $K_{m,n}$ be the complete bipartite graph with $(m+n)$ vertices. Clearly, the eccentricity of the vertices of $K_{m,n}$ are equal to two; and there are m number of vertices of degree n and n number of vertices of degree m. So the connective eccentricity index of $K_{m,n}$ is mn. Let

$K^*_{m,n}$ be the thorny complete bipartite graph obtained from $K_{m,n}$ by attaching a number of pendent vertices to each vertex of $K_{m,n}$. Then we get the following result.

Theorem 3.1. The connective eccentricity index of $K^*_{m,n}$ is given by

$$C^{\xi}(K^*_{m,n}) = C^{\xi}(K_{m,n}) + \frac{7}{12}T.$$

Proof: Let the vertex set of $K_{m,n}$ is given by $\{v_1, v_2, ..., v_m, u_1, u_2, ..., u_n\}$ and $p_i,\ i = 1, 2, ..., m$ and $p'_i,\ i = 1, 2, ..., n$ be the number of pendent vertices attached to v_i and u_i respectively to obtain $K^*_{m,n}$. Let, the newly attached pendent vertices are denoted by v_{ij}, $i = 1, 2, ..., m;\ j = 1, 2, ..., p_i$ and u_{ij}, $i = 1, 2, ..., n;\ j = 1, 2, ..., p'_i$. Then the degree and eccentricity of the vertices of $K^*_{m,n}$ are given by $d_{K^*_{m,n}}(v_i) = n + p_i$, $d_{K^*_{m,n}}(v_{ij}) = 1$, $\varepsilon_{K^*_{m,n}}(v_i) = 3$, $\varepsilon_{K^*_{m,n}}(v_{ij}) = 4$, for $i = 1, 2, ..., m$; $j = 1, 2, ..., p_i$ and $d_{K^*_{m,n}}(u_i) = m + p'_i$, $d_{K^*_{m,n}}(u_{ij}) = 1$, $\varepsilon_{K^*_{m,n}}(u_i) = 3$, $\varepsilon_{K^*_{m,n}}(u_{ij}) = 4$, for $i = 1, 2, ..., n; j = 1, 2, ..., p'_i$. Thus, the connective eccentricity index of $K^*_{m,n}$ is given by

$$\begin{aligned} C^{\xi}(K^*_{m,n}) &= \sum_{i=1}^{m} \frac{d_{K^*_{m,n}}(v_i)}{\varepsilon_{K^*_{m,n}}(v_i)} + \sum_{i=1}^{n} \frac{d_{K^*_{m,n}}(u_i)}{\varepsilon_{K^*_{m,n}}(u_i)} + \sum_{i=1}^{m}\sum_{j=1}^{p_i} \frac{d_{K^*_{m,n}}(v_{ij})}{\varepsilon_{K^*_{m,n}}(v_{ij})} + \sum_{i=1}^{n}\sum_{j=1}^{p'_i} \frac{d_{K^*_{m,n}}(u_{ij})}{\varepsilon_{K^*_{m,n}}(u_{ij})} \\ &= \sum_{i=1}^{m} \frac{n + p_i}{2} + \sum_{i=1}^{n} \frac{m + p'_i}{2} + \sum_{i=1}^{m}\sum_{j=1}^{p_i} \frac{1}{4} + \sum_{i=1}^{n}\sum_{j=1}^{p'_i} \frac{1}{4} \\ &= \frac{mn}{3} + \frac{mn}{3} + \frac{1}{3}\left(\sum_{i=1}^{m} p_i + \sum_{i=1}^{n} p'_i\right) + \frac{1}{4}\left(\sum_{i=1}^{m} p_i + \sum_{i=1}^{n} p'_i\right) \end{aligned}$$

from where we get the desired result . □

4. The Thorny star

Let, $S_n = K_{1,(n-1)}$ be the star graph on *n* vertices. Clearly, $C^{\xi}(S_n) = \frac{3}{2}(n-1)$. Let S^*_n be the thorny graph obtained by joining p_i, to each vertex of v_i, *i=1,2,…n.*

Theorem 4.1. The connective eccentricity index of thorn star S^*_n is given by

$$C^{\xi}(S^*_n) = \frac{5}{6}(n-1) + \frac{7}{12}T + \frac{p_1}{4}$$

where p_1 is the number of pendent vertices added to the central vertex of S_n.

Proof: Let S^*_n be the thorny graph of S_n obtained by attaching p_i pendent vertices to each vertex v_i ($i = 1, 2, ..., n$) of S_n so that the degree and eccentricity of the vertices of

S_n^* are given by $d_{S_n^*}(v_1) = n-1+p_1$, $d_{S_n^*}(v_i) = 1+p_i$, $i = 2,...,n$, $d_{S_n^*}(v_{ij}) = 1$, $\varepsilon_{S_n^*}(v_1) = 2$, $\varepsilon_{S_n^*}(v_i) = 3$, $i = 2,...,n$, $\varepsilon_{S_n^*}(v_{ij}) = 4$, for $i = 2,...,n; j = 1,2,..., p_i$, $\varepsilon_{S_n^*}(v_{1j}) = 3$ for $j = 1,2,..., p_1$. So the connective eccentricity index of S_n^* is given by

$$C^{\xi}(S_n^*) = \sum_{i=1}^{n} \frac{d_{S_n^*}(v_i)}{\varepsilon_{S_n^*}(v_i)} + \sum_{i=1}^{n}\sum_{j=1}^{p_i} \frac{d_{S_n^*}(v_{ij})}{\varepsilon_{S_n^*}(v_{ij})}$$

$$= \frac{d_{S_n^*}(v_1)}{\varepsilon_{S_n^*}(v_1)} + \sum_{i=2}^{n} \frac{d_{S_n^*}(v_i)}{\varepsilon_{S_n^*}(v_i)} + \sum_{j=1}^{p} \frac{d_{S_n^*}(v_{1j})}{\varepsilon_{S_n^*}(v_{1j})} + \sum_{i=2}^{n}\sum_{j=1}^{p_i} \frac{d_{S_n^*}(v_{ij})}{\varepsilon_{S_n^*}(v_{ij})}$$

$$= \frac{n-1+p_1}{2} + \sum_{i=2}^{n} \frac{1+p_i}{3} + \sum_{j=1}^{p_i} \frac{1}{3} + \sum_{i=2}^{n}\sum_{j=1}^{p_i} \frac{1}{4}$$

$$= \frac{n-1}{2} + \frac{p_1}{2} + \frac{1}{3}\sum_{i=2}^{n} p_i + \frac{n-1}{3} + \frac{p_1}{3} + \frac{1}{4}\sum_{i=2}^{n} p_i$$

$$= \frac{5}{6}(n-1) + \frac{1}{3}\sum_{i=1}^{n} p_i + \frac{n-1}{3} + \frac{p_1}{2} - \frac{p_1}{4} + \frac{1}{4}\sum_{i=1}^{n} p_i$$

from where the desired result follows. □

5. The Thorny cycle

Let C_n be the cycle with vertex set $\{v_1, v_2, ..., v_n\}$. The thorny cycle C_n^* is obtained from C_n by attaching p_i, thorns at every vertex of C_n, *i=1,2,…n.*

Theorem 5.1. The connective eccentricity index of the thorny cycle C_n^* is given by

$$C^{\xi}(C_n^*) = \begin{cases} \dfrac{4T(n+3)}{(n+2)(n+4)} + \dfrac{4n}{(n+2)}, & if\ n\ is\ even \\ \dfrac{4T(n+2)}{(n+1)(n+3)} + \dfrac{4n}{(n+1)}, & if\ n\ is\ odd. \end{cases}$$

Proof: The degree and eccentricity of the vertices of C_n^* are given by $d_{C_n^*}(v_i) = p_i + 2$, $d_{C_n^*}(v_{ij}) = 1$, $\varepsilon_{C_n^*}(v_i) = \frac{n+1}{2}$, if n is odd and $\varepsilon_{C_n^*}(v_i) = \frac{n+2}{2}$, if n is even; $\varepsilon_{C_n^*}(v_{ij}) = \frac{n+3}{2}$, if n is odd and $\varepsilon_{C_n^*}(v_{ij}) = \frac{n+4}{2}$, if n is even; for $i = 1,2,...,n; j = 1,2,..., p_i$. So, when n is an odd number, the connective eccentricity index of C_n^* is given by

$$C^{\xi}(C_n^*) = \sum_{i=1}^{n} \frac{d_{C_n^*}(v_i)}{\varepsilon_{C_n^*}(v_i)} + \sum_{i=1}^{n}\sum_{j=1}^{p_i} \frac{d_{C_n^*}(v_{ij})}{\varepsilon_{C_n^*}(v_{ij})}$$

$$=\sum_{i=1}^{n}\frac{p_i+2}{(n+1)/2}+\sum_{i=1}^{n}\sum_{j=1}^{p_i}\frac{1}{(n+3)/2}$$

$$=\frac{2}{(n+1)}\sum_{i=1}^{n}(p_i+2)+\frac{2}{(n+3)}\sum_{i=1}^{n}\sum_{j=1}^{p_i}1$$

$$=\frac{2}{(n+1)}(T+2n)+\frac{2T}{(n+3)}$$

from where we get the desired result. Proceeding similarly, if n is an even integer, the desired result follows. □

6. Thorny path

Let P_m denote a path graph on m vertices. The thorny graph of path graph is denoted by P_m^* and is obtained by attaching a number of degree one vertices to every vertex of P_m. In the following we find connective eccentricity index of P_m^*.

Theorem 6.1. The connective eccentricity index of P_m^* is given by

$$C^{\xi}(P_m^*)=\begin{cases}\sum_{i=0}^{n-1}\frac{p_i+p_i'+4}{n+i+2}+\sum_{i=0}^{n}\frac{p_i+p_i'}{n+i+3}+\frac{p_i+p_i'+2}{2n+2}, when\, m=2n+2\\ \sum_{i=1}^{n-1}\frac{p_i+p_i'+4}{n+i+1}+\frac{p_0+2}{n+1}+\frac{p_0'}{n+2}+\sum_{i=1}^{n-1}\frac{p_i+p_i'}{n+i+2}+\frac{p_i+p_i'+2}{2n+1}, when\, m=2n+1.\end{cases}$$

.

Proof: When the number of vertices of P_m is even, say $m=2n+2$, let the vertices of P_m are consecutively denoted by $v_n', v_{n-1}', \ldots, v_2', v_1', v_0', v_0, v_1, v_2, \ldots, v_{n-1}, v_n$ where v_0' and v_0 are the centers of the path P_{2n+2} with eccentricity ($n+2$). We attach p_i and p_i' number of pendent vertices to each v_i and v_i' respectively ($i=1,2,\ldots n$). Then the degree and eccentricity of the other vertices of P_{2n+2}^* is given by $d_{P_m^*}(v_n)=1+p_n$, $d_{P_m^*}(v_i)=2+p_i$, $d_{P_m^*}(v_n')=1+p_n'$, $d_{P_m^*}(v_i')=2+p_i'$, $i=0,1,\ldots,n-1$, $d_{P_m^*}(v_{ij})=1=d_{P_m^*}(v_{ij}')$ for $i=0,1,\ldots,n$ and $j=1,2,\ldots,p_i$, $\varepsilon_{P_m^*}(v_i)=n+i+2=\varepsilon_{P_m^*}(v_i')$, for $i=0,1,\ldots,n$, $\varepsilon_{P_m^*}(v_{ij})=n+i+3=\varepsilon_{P_m^*}(v_{ij}')$ for $i=0,1,\ldots,n; j=1,2,\ldots,p_i$. So the connective eccentricity index of P_{2n+2}^* is given by

$$C^{\xi}(P_m^*)=C_1^{\xi}(P_m^*)+C_2^{\xi}(P_m^*)$$

where, $C_1^{\xi}(P_m^*)=\sum_{i=0}^{n}\frac{d_{P_m^*}(v_i)}{\varepsilon_{P_m^*}(v_i)}+\sum_{i=0}^{n}\frac{d_{P_m^*}(v_i')}{\varepsilon_{P_m^*}(v_i')}$

$$=\sum_{i-0}^{n-1}\frac{2+p_i}{n+2+i}+\frac{1+p_n}{2n+2}+\sum_{i-0}^{n-1}\frac{2+p_i'}{n+2+i}+\frac{1+p_n'}{2n+2}$$

$$=\sum_{i=0}^{n-1}\frac{p_i+p_i'+4}{n+2+i}+\frac{p_n+p_n'}{2n+2}.$$

Also, $C_2^{\xi}(P_m^*)=\sum_{i=1}^{n}\sum_{j=1}^{p_i}\frac{d_{P_m^*}(v_{ij})}{\varepsilon_{P_m^*}(v_{ij})}+\sum_{i=1}^{n}\sum_{j=1}^{p_i'}\frac{d_{P_m^*}(v_{ij}')}{\varepsilon_{P_m^*}(v_{ij}')}$

$$=\sum_{i=1}^{n}\sum_{j=1}^{p_i}\frac{1}{n+i+3}+\sum_{i=1}^{n}\sum_{j=1}^{p_i'}\frac{1}{n+i+3}$$

$$=\sum_{i=0}^{n}\frac{p_i+p_i'}{n+i+3}.$$

Combining, the desired result follows. Again, if the number of vertices of P_m is odd, say *m=2n+1*, then in a similar fashion we get the desired result. □

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