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Multi-objective Cost Varying Transportation Problem using Fuzzy Programming

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Abstract. In this paper we represent a multi-objective transportation problem whose transportation cost is varying due to capacity of 2 -vehicles as well as transport quantities. The 2 -vehicle multi-objective cost varying transportation problem is a Bi-level Mathematical programming model. To solve this model, use north west corner rule for determining initial basic feasible solution and then set up unit cost (which varies in each iteration) for each cost matrix corresponding to each objective by proper choice of vehicles with our proposed algorithm. Apply optimality test for determining optimal solution for each objective separately. Then apply fuzzy programming technique to sole this model. Numerical examples are presented to illustrate the model.

Keywords: Transportation Problem, Basic Cell, Non-basic Cell, North West Corner Rule

AMS Mathematics Subject Classification (2010): 90B

1. Introduction

Transportation problem is a special class of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. in order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may be vary due to capacity of vehicles which are transport the commodities from sources to destinations according their demands.

In this paper, we present the 2 -vehicle multi-objective cost varying transportation problem which is a Bi-level Mathematical programming model. To solve this model, use north west corner rule for determining initial basic feasible solution and then set up unit cost (which varies in each iteration) for each cost matrix corresponding to each objective by proper choice of vehicles with our proposed algorithm. Apply optimality test for determining optimal solution for each objective separately. Then apply fuzzy programming technique to solve this model.

2. Mathematical formulation

A multi-objective transportation problem can be stated in **Model 1** as follows:

Model 1

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, r = 1,\dots,k$$

$$subject\ to \quad \sum_{i=1}^{m} x_{ij} = a_i,\ i = 1,\dots,m \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = b_j,\ j = 1,\dots,n \tag{2}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j;\ x_{ij} \geq 0\ \forall i,\ \forall j$$

2.2. 2-Vehicle cost varying transportation problem

Suppose there are two types off vehicles V_1, V_2 from each source to each destination. Let C_1 and $C_2(> C_1)$ are the capacities(in unit) of the vehicles V_1 and V_2 respectively. $R_{ij}^{r} = (R1_{ij}^{r}, R2_{ij}^{r}), r = 1,\dots,k$ represent transportation cost for each cell (i, j); where $R1_{ij}^{r}, r = 1,\dots,k$ are the transportation cost from source $O_i, i = 1,\dots,m$ to the destination $D_j, j = 1,\dots,n$ by the vehicle V_1. And $R2_{ij}^{r}, r = 1,\dots,k$ are the transportation cost from source $O_i, i = 1,\dots,m$ to the destination $D_j, j = 1,\dots,n$ by the vehicle V_2. So, for each $r = 1,\dots,k$ cost varying transportation problem can be represent in the following tabulated form.

	D_1	D_2	..	D_n	*stock*
O_1	c_{11}^{r} $R1_{11}^{r}, R2_{11}^{r}$	$c_{12}^{r},$ $R1_{12}^{r}, R2_{12}^{r}$	....	$c_{1n}^{r},$ $R1_{1n}^{r}, R2_{1n}^{r}$	a_1
O_2	c_{21}^{r} $R1_{21}^{r}, R2_{21}^{r}$	$c_{22}^{r},$ $R1_{22}^{r}, R2_{22}^{r}$	....	$c_{2n}^{r},$ $R1_{2n}^{r}, R2_{2n}^{r}$	a_2
....	....	....	....	....	....
O_m	c_{n1}^{r} $R1_{m1}^{r}, R2_{m1}^{r}$	$c_{n2}^{r},$ $R1_{m2}^{r}, R2_{m2}^{r}$	....	$c_{nn}^{r},$ $R1_{mn}^{r}, R2_{mn}^{r}$	a_m
Demand	b_1	b_2	....	b_n	

Table 1: 2-Vehicle multi-objective cost varying transportation problem.

where $c_{ij}^{r} i = 1,\ldots,m; j = 1,\ldots,n; r = 1,\ldots,k$ are not constants.

2.2.1. Algorithm(CVTP)

Step 1. Since unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij}^{r} by North-west corner rule, for basic cell we determine c_{ij}^{r} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij}^{r} = \begin{cases} \dfrac{p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r}}{x_{ij}}, & if \quad x_{ij} \neq 0 \\ & \qquad\qquad r = 1,\ldots,k \\ 0 & if \quad x_{ij} = 0 \end{cases} \tag{3}$$

$$where \ p1_{ij}, p2_{ij}, i = 1,\ldots,m; j = 1,\ldots,n \ are \ integer \ solution \ of$$

$$\min p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r}, \quad s.t. \ x_{ij} \leq p1_{ij} C_1 + p2_{ij} C_2$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}^{r}, then for non-basic cell c_{ij}^{r} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij}^{r} = \begin{cases} \dfrac{p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r}}{x_{ij}}, & if \quad x_{ij} \neq 0 \\ & \qquad\qquad r = 1,\ldots,k \\ 0 & if \quad x_{ij} = 0 \end{cases} \tag{4}$$

$$where \ p1_{ij}, p2_{ij}, i = 1,\ldots,m; j = 1,\ldots,n \ are \ integer \ solution \ of$$

$$\min p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r}$$

$$s.t. \ x_{ij} \leq p1_{ij} C_1 + p2_{ij} C_2$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij}^{r} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c_{ij}^{r}, r = 1,\ldots,k$ by **Step 2** and for non-basic we fix $c_{ij}^{r}, r = 1,\ldots,k$ by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

2.2.2. Bi-level mathematical programming for 2-vehicle multi-objective cost varying transportation problem

The Bi-level mathematical programming for 2-vehicle multi-objective cost varying transportation problem is formulated in **Model 2** as follows:

Model 2

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, r = 1,\ldots,k \tag{5}$$

where, c_{ij}^{r} *is determined by following mathematical programming*

$$c_{ij}^{r} = \begin{cases} \dfrac{p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r}}{x_{ij}}, & if \quad x_{ij} \neq 0 \\ & \qquad\qquad r = 1,\ldots,k \\ 0 & if \quad x_{ij} = 0 \end{cases}$$

$$\min p1_{ij} R1_{ij}^{r} + p2_{ij} R2_{ij}^{r} \tag{6}$$

$$s.t. \;\; x_{ij} \le p1_{ij} C_1 + p2_{ij} C_2$$

$$\sum_{i=1}^{m} x_{ij} = a_i, \; i = 1,\ldots,m, \quad \sum_{j=1}^{n} x_{ij} = b_j, \; j = 1,\ldots,n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \;\; x_{ij} \ge 0 \;\; \forall i, \; \forall j$$

$$where \;\; p1_{ij}, \, p2_{ij}, i = 1,\ldots,m; \, j = 1,\ldots,n \;\; are \; integer \; solution \; of$$

2.3. Solution procedure of 2-Vehicle Multi-objective Cost Varying Transportation Problem(TVMOCMTP).

To solve **Model 2** first we determine $x_{ij}^{r}, r-1,\ldots,k$ for basic each cell (i, j) then, determined $c_{ij}^{r}, r-1,\ldots,k$ for basic each cell (i, j) and for non-basic cell (i,j) (by possible allocation of loop) through **Algorithm)** with the help of C-programming then TVMOCMTP converted in linear programming **Model 3** with varying unit cost c_{ij}^{r} as follows:

2.3.1. Linear Programming Formulation of TVMOCMTP

A TVMOCMTP can be stated in **Model 3** as follows:

Model 3

$$\min Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, r = 1,\ldots,k \tag{7}$$

$$subject\ to \quad \sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,\ldots,m;\ \sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,\ldots,n;$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j;\ x_{ij} \geq 0\ \forall i,\ \forall j$$

The subscript on Z_r and superscript on c_{ij}^r denote the r^{th} varying unit cost.

Using a linear membership function, the crisp model can be simplified in **Model 4** as follows:

Model 4

$$\max \lambda \tag{8}$$

$$subject\ to \quad Z_r + \lambda(U_r - L_r) \leq U_r,\ r = 1,\ldots,k$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,\ldots,m;\ \sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,\ldots,n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j;\ x_{ij} \geq 0\ \forall i,\ \forall j, \lambda \geq 0$$

where U_r and L_r are the maximum and minimum value of Z_r, $r = 1,2.....,k$

3. Numerical example

Consider a 2-vehicle cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	5,7	4,6	8,10	14
O_2	2,3	6,8	7,9	16
O_3	3,4	10,12	4,6	12
Demand	10	15	17	

Table 3.1:

	D_1	D_2	D_3	*stock*
O_1	10,12	11,13	12,14	14
O_2	8,10	6,9	5,7	16
O_3	15,18	14,16	4,6	12
Demand	10	15	17	

Table 3.2:

Using our proposed **Algorithm ,** if we consider **Table-3.1** as a single objective 2-vehicle cost varying transportation problem then optimal solution is given by

$$x^1 = \{x_{11}^1 = 4, x_{12}^1 = 10, x_{13}^1 = 0, x_{21}^1 = 6, x_{22}^1 = 0, x_{23}^1 = 10, x_{31}^1 = 0, x_{32}^1 = 0, x_{33}^1 = 7\}$$

$$c_{11}^1 = \frac{5}{4},\ c_{12}^1 = \frac{4}{10},\ c_{13}^1 = \frac{8}{4},\ c_{21}^1 = \frac{1}{3}^1,\ c_{22}^1 = 1,\ c_{23}^1 = \frac{7}{10},\ c_{31}^1 = \frac{1}{2},\ c_{32}^1 = \frac{10}{7},\ c_{33}^1 = \frac{4}{7}$$

Again using our proposed **Algorithm,** if we consider **Table-3.2** as a single objective 2-vehicle cost varying transportation problem then optimal solution is given by

$$x^2 = \{x_{11}^2 = 5, x_{12}^2 = 0, x_{13}^2 = 70, x_{21}^2 = 45, x_{22}^2 = 15, x_{23}^2 = 0, x_{31}^2 = 0, x_{32}^2 = 40, x_{33}^2 = 0\}$$

$$c_{11}^2 = 1,\ c_{12}^2 = \frac{11}{4},\ c_{13}^2 = \frac{12}{4},\ c_{21}^2 = \frac{4}{3},\ c_{22}^2 = 1,\ c_{23}^2 = \frac{1}{2},\ c_{31}^2 = \frac{15}{7},\ c_{32}^2 = \frac{7}{6},\ c_{33}^2 = \frac{4}{7}$$

Also $Z_1(x^1) = 22$, $Z_1(x^2) = 31.1$, $Z_2(x^1) = 48.5$, $Z_2(x^2) = 36$

Solve by **Model 4** by Lingo-13 package. The optimal solution of the problem is $x^* = \{x_{11} = 6.849294, x_{12} = 7.150706, x_{13} = 0, x_{21} = 3.150706, x_{22} = 7.84929, x_{23} = 5.0, x_{31} = 0, x_{32} = 0, x_{33} = 12\}$

$Z_1^* = 30.67857$ $Z_2^* = 47.92111$ $\lambda^* = 0.0463$

4. Conclusion

In this paper we have developed two-vehicle multi-objective cost varying transportation problem. We transfer this multi-objective cost varying transportation problem to usual multi-objective transportation problem by Northwest corner rule and then apply optimality test where unit transportation cost vary from one table to another table. This problem is more real life problem than usual transportation problem.

REFERENCES

1. F.L.Hitchcock, The distribution of a product from several sources to numerouslocalities, *Journal of Mathematical Physics*, 20(17) (1941) 224–230.
2. G.B.Dantzig, *Linear Programming and Extensions*, Princeton University Press, Princeton, (1963).
3. H.A.Taha, *Operation Research: An Introduction*, Macmillan, New York, (1992).
4. H.J.Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, 91(1) (1997) 37–43.
5. H.J.Zimmermann, *Fuzzy Set Theory and its Applications*, 4th ed., Kluwer-Nijhoff, Boston, (2001).
6. A.Panda and C.B.Das, 2-vehicle cost varying transportation problem, *Journal of Uncertain Systems*, 8(1) (2014) 44–57.
7. A.Panda and C.B.Das, *Capacitated Transportation Problem under Vehicles*, LAPLAMBERT Academic Publisher, Deutschland/Germany, (2014).
8. A.Panda and C.B.Das, *N*-vehicle cost varying transportation problem, *Advanced Modeling and Optimization*, 15(3) (2013) 583–610 .
9. A.Panda, C.B.Das, Cost varying interval transportation problem under 2-vehicle, *Journal of New Results in Sciences*, 3(3) (2013) 19–36.
10. A.Panda and C.B.Das, *N*-vehicle cost varying capacitated transportation problem, *Ciit-international Journal of Fuzzy Systems*, 5(7) (2013) 191-204.
11. R.Verma, M.P.Biswal and A.B.Verma, Fuzzy programming technique to solve multi-objective transportation problems with some non-linear functions, *Fuzzy Sets and Systems*, 91(1) (1997) 37–43.