Annals of Pure and Applied Mathematics
Vol. 8, No. 2, 2014, 153-159
ISSN: 2279-087X (P), 2279-0888(online)
Published on 17 December 2014
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Conflict-Free Coloring of Extended Duplicate Graph of Twig

E.Esakkiammal, K. Thirusangu **and** ***S. Bala***

Department of Mathematics
S.I.V.E.T. College, Gowrivakkam, Chennai-600 073, India
E-Mail :esakkiammal2682@gmail.com, kthirusangu@gmail.com,
balasankarasubbu@yahoo.com

Received 1 November 2014; accepted 21 November 2014

Abstract. Graph coloring is one of the most important area of research in graph theory. Conflict-free coloring is defined as a vertex coloring such that for each vertex $v \in$ V,there exists a vertex in the neighbourhood of v denoted by N(v),whose color is different from color of each other vertex in N[v].In this paper, we prove the existence of conflict-freevertex coloring in EDG(T_m) andshow that χ_{CF}(EDG(T_m)) = Δ+ 1. As a consequence, we show that the induced subgraphof each color class with s vertices is $\overline{K_s}$, where $\overline{K_s}$ is the complement of the complete graph K_s .

Keywords: Conflict-free coloring, extended duplicate graph of twig graphs.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The vertex coloring of a graph is coloring the vertices of the graph in such a way that adjacent vertices have different colors. Motivated by the problem of frequency assignments in cellular networks, Even et.al [3] and Smorodinsky [6] introduced the concept of conflict-free coloring. Pach and Tarados [5] instituted the idea of conflict coloring through graphs and hyper graphs. Glebov et.al [4] further brought forwarded the concept of conflict-free coloring through simple graphs. The Extended Duplicate Graph of Twig graphs was introduced by Thirusanguetal [7]. Other kind of labelling are studied in [8-13].

In this paper, we prove the existence of conflict-free vertex coloring in Extended Duplicate Graph of Twig graphs EDG(T_m) which yields the result that the induced subgraph ofeach color classwith s vertices is $\overline{K_s}$, where $\overline{K_s}$ is the complement of the complete graph K_s .It is also proved that χ_{CF}(EDG(T_m)) = Δ + 1.

2. Preliminaries

In this section we give the basic notions relevant to this paper.

Definition 2.1. A graph G(V,E) obtained from a path by joining exactly two pendent edges to each internal vertices of a path is called a Twig graph. A Twig graph with *m*internal vertices is denoted by T_m .

Definition 2.2. Let G(V,E) be a simple graph. A duplicate graph of G is DG=(V_1 , E_1) where the vertex set V_1 =V ∪ V′ and V ∩ V′ = φ and f:V→V′ is bijective and the edgeset E_1 of DG is defined as follows: The edge*uv* is in E if and only if both*uv*′and $u'v$ are edges in E_1.

Definition 2.3. Let DG=(V_1 , E_1) be a duplicate graph of the Twig graph G(V,E). We add an edge between any one vertex from V to any other vertex in V′ except the terminal vertices of V and V′. For our convenience, we take v_2 ∈V and v_2' ∈ V′ and thus the edge v_2v_2' is formed. We call this new graph as the extended duplicate graph of the Twig graph T_m and is denoted by EDG(T_m). The vertex set and the edge set of EDG(T_m) are given as follows :

V={v_1 , v_2 , ..., v_{3m+2}, v_1', v_2',…………., v_{3m+2}' }

E ={ v_1v_2', $v_1'v_2$, v_2v_2' }∪ { $v_s'v_l$ ∪ $v_{s+3}v_{l+3}'$ ∪ $v_s v_l'$ ∪ $v_{s+3}'v_{l+3}$/s=2+6i ,l = s+j+1; 0≤ $i \leq \left[\frac{m-2}{2}\right]$; 0 ≤ j ≤ 2 } if m is even

E= {v_1v_2', $v_1'v_2$, v_2v_2'}∪{ $v_s'v_l$ ∪ $v_s v_l'$/ s=2+6i , l = s+j+1;

0≤ $i \leq \left[\frac{m-1}{2}\right]$; 0 ≤ j ≤ 2 } ∪ { $v_{s+3}'v_{l+3}$ ∪ $v_{s+3}v_{l+3}'$/s=2+6i , = s+j+1; 0≤ $i \leq \left[\frac{m-3}{2}\right]$; 0 ≤ j ≤2 } if m is odd

Clearly EDG(T_m) has 6*m*+4 vertices and 6*m*+3 edges where *m*is the number of internal vertices of the Twig graph.

Definition 2.4. Let G=(V,E) be a simple graph. For every vertex *u*∈ V, we denote N(*u*) ={*v* ∈V: *uv*∈ E}, its neighbourhood and by N(*u*) ∪ {*u*}=N[*u*], its closed neighbourhood.Also the maximum degree of the graph is denoted by Δ.

Definition 2.5. A vertex coloring χ of G is called conflict-free if for each vertex *u*∈ V, there exists a vertex *v*in N(*u*) whose color is different from the color of each other vertex in N[*u*].

The conflict-free chromatic number χ_{CF}(G) is the smallest r, such that there exists a conflict-free r-coloring of G.

Definition 2.6. For any set S of vertices of graph G, the induced subgraph<S>, is the maximal subgraph of G with vertex set S. Thus two vertices are adjacent in S if and only if they are adjacent in G.

Definition 2.7. A graph G is said to be a complete graph if each pair of vertices is adjacent.A complete graph of *n* vertices is denoted by K_n .

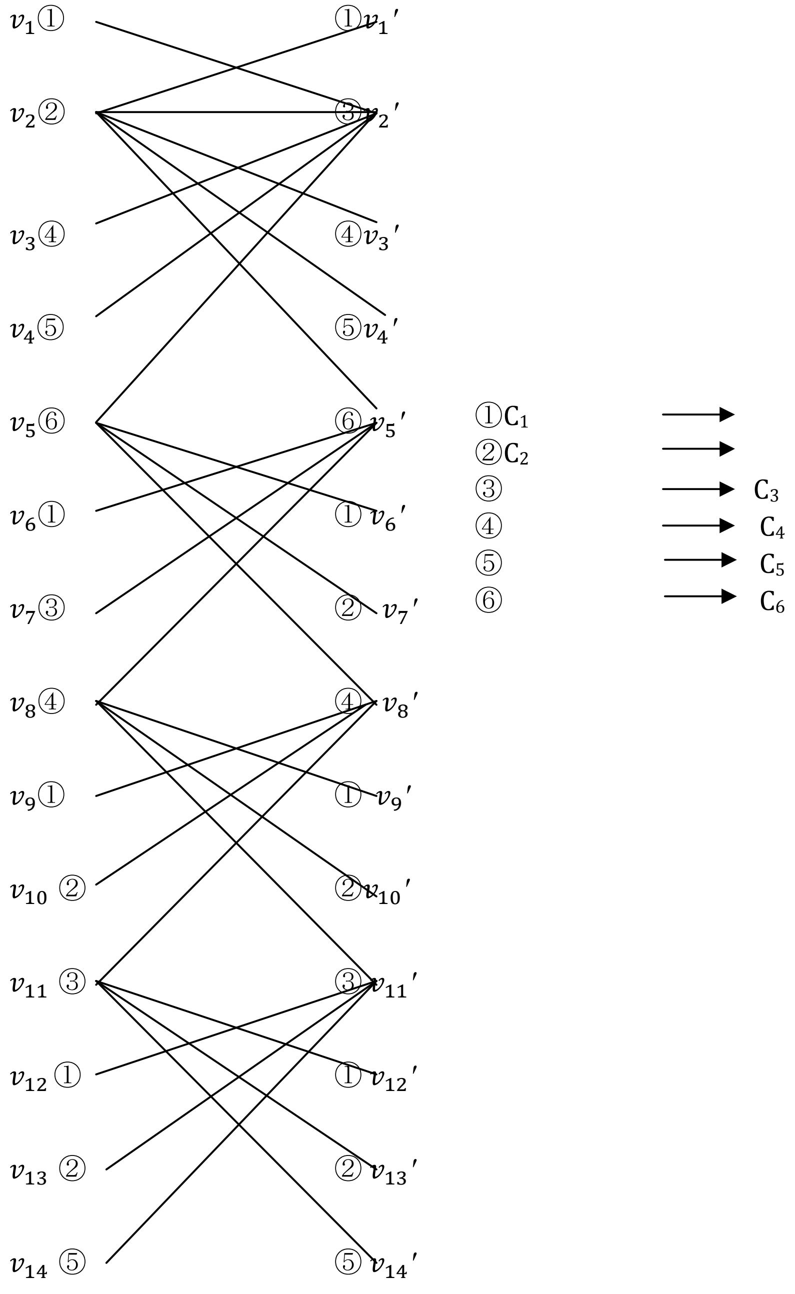
The complement$\bar{G}$ of G is defined to be the graph which has V as its vertex set and two veritices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

3. Main results

In this section, we present an algorithm to obtain the conflict-free coloring of EDG(T_m).We prove that EDG(T_m) admits conflict-free coloring. We also show that the induced subgraph of each of the color class is a totally disconnected graph. Also we find the conflict-free chromatic number of EDG(T_m) is Δ + 1.

Algorithm 3.1:
Procedure: Conflict-free coloring of EDG(T_m)
Input:V←{v_1, v_2, ..., v_{3m+2}, v_1', v_2',…………., v_{3m+2}' }
E←{e_1, e_2, ..., e_{3m+2}, e_1', e_2',…………., e_{3m+1}' }
Output: Conflict-free colored EDG(T_m) graph.
if m=1
$v_1 \cup v_1' \leftarrow C_1$
$v_2 \leftarrow C_2$
$v_2' \leftarrow C_3$
$v_3 \cup v_3' \leftarrow C_4$
else
if m=2
$v_1 \cup v_1' \cup v_6 \cup v_6' \leftarrow C_1$
$v_2 \cup v_7' \leftarrow C_2$
$v_2' \cup v_7 \leftarrow C_3$
$v_3 \cup v_3' \cup v_8 \cup v_8' \leftarrow C_4$
else
for n= 2 to m do
$v_1 \cup v_1' \cup (\bigcup_{i=2}^{n} v_{3i} \cup v_{3i}') \leftarrow C_1$
$v_2 \cup v_7' \cup (\bigcup_{i=3}^{n} v_{3i+1} \cup v_{3i+1}') \leftarrow C_2$
$v_2' \cup v_7 \cup (\bigcup_{i=3k/k\in N}^{n} v_{3i+2} \cup v_{3i+2}') \leftarrow C_3$
$v_3 \cup v_3' \cup (\bigcup_{i=3k-1/k\in N}^{n} v_{3i+2} \cup v_{3i+2}') \leftarrow C_4$
end for
end if
end if
if (m≤ 3)
$v_4 \cup v_4' \leftarrow C_5$
else
$v_4 \cup v_4' \cup (\bigcup_{i=3k+1/k\in N}^{m} v_{3i+2} \cup v_{3i+2}') \leftarrow C_5$
end if
$v_5 \cup v_5' \leftarrow C_6$
end procedure

Example 3.1. Conflict-free colored EDG(T_4)graph.



Here C_i foreach i represent the color, $1 \leq i \leq 6$.

Figure 1:

Theorem 3.1. EDG(T_m) is a conflict-free colorable graph.
Proof: The EDG(T_m) has 6m+4 vertices and 6m+3 edges.
To prove EDG(T_m) is a conflict-free colorable graph , it is enough to prove that the vertex of maximum degree , either v_2 or $v_2{}'$ is conflict-free colorable.
Using Algorithm 3.1, the color class C_2 has vertices
$v_2, v_7', v_{10}, v_{10}', v_{13}, v_{13}', \ldots, v_{3m+1}, v_{3m+1}'$. Thus $v_2 \in C_2$. Also the neighbours of v_2 are $\{v_1{}', v_2{}', v_3{}', v_4{}', v_5{}'\}$ where $v_1{}' \in C_1$, $v_2{}' \in C_3$, $v_3{}' \in C_4$, $v_4{}' \in C_5$, $v_5{}' \in C_6$. That is each neighbours of v_2 is in different color classes.
Similarly for all the vertices of EDG(T_m) in a color class has neighbours in different color classes .

Hence EDG(T_m) is a conflict-free colorable graph.

Theorem 3.2. In Extended Duplicate Graph EDG(T_m), the induced subgraph of each color class with $n_i$vertices is totally disconnected graph $\overline{K}_{n_i}$ where $\overline{K}_{n_i}$ is the complement of thecomplete graphK_{n_i}.
Proof: Denote the vertex set of EDG(T_m) as V = $\{v_1, v_2, \ldots\ldots, v_{3m+2}, v_1{}', v_2{}' \ldots\ldots\ldots, v_{3m+2}{}'\}$
Using algorithm, the colors in EDG(T_m) are classified as follows :
The color class

$$C_1 = \begin{cases} v_1 \cup v_1' \text{if } m = 1 \\ v_1 \cup v_1{}' \cup (\cup_{i=2}^{n} v_{3i} \cup v_{3i}{}') \text{ if } 2 \leq n \leq m \end{cases}$$

The color class

$$C_2 = \begin{cases} v_2 \text{if } m = 1 \\ v_2 \cup v_7' \text{if } m = 2 \\ v_2 \cup v_7{}' \cup (\cup_{i=3}^{n} v_{3i+1} \cup v_{3i+1}{}') \text{ if } 3 \leq n \leq m \end{cases}$$

The color class

$$C_3 = \begin{cases} v_2{}' \quad \text{if } m = 1 \\ v_2{}' \cup v_7 \text{ if } m = 2 \\ v_2{}' \cup v_7 (\cup_{i=3k/k\in N}^{n} v_{3i+2} \cup v_{3i+2}{}') \text{ if } 3 \leq n \leq m \end{cases}$$

The color class$C_4 = \begin{cases} v_3 \cup v_3' \text{if } m = 1 \\ v_3 \cup v_3{}' \cup (\cup_{i=3k-1/k\in N}^{n} v_{3i+2} \cup v_{3i+2}{}') \text{ if } 2 \leq n \leq m \end{cases}$

The color class$C_5 = \begin{cases} v_4 \cup v_4' \text{if } m \leq 3 \\ v_4 \cup v_4{}' \cup (\cup_{i=3k+1/k\in N}^{n} v_{3i+2} \cup v_{3i+2}{}') \text{ if } 4 \leq n \leq m \end{cases}$

The colorclass C_6 = $v_5 \cup v_5' \ \forall m$.

Let $n_i$be the number of colors in C_i where 1≤i≤ 6 respectively in the EDG(T_m) graph.
Thus the vertices in the induced subgraph obtained from each of the color classes are not adjacent, since they are not adjacent in EDG(T_m).

Hence the induced subgraph of $n_i$vertices relative to each of the above color class is a totally disconnected graph$\overline{K}_{n_i}$, where $\overline{K}_{n_i}$ is the complement of the complete graph K_{n_i}, 1 ≤i ≤ 6.

Theorem 3.3. The conflict-free chromatic number of EDG(T_m) is Δ + 1.
Proof: Consider the EDG(T_m) which has 6m+4 vertices. Denote them as

$$V = \{v_1, v_2, \ldots, v_{3m+2}, v_1', v_2', \ldots, v_{3m+2}'\} = V_1 \cup V_2$$

where $V_1 = \{v_1, v_2, v_3, v_4, v_5, v_1', v_2', v_3', v_4', v_5'\}$ and $V_2 = V \setminus V_1$.

Case (i) : m = 1

Consider the vertex set V_1 of EDG(T_1). Since deg(v_2) = deg(v_2'),| $N(v_2)$|=| $N(v_2')$| =Δ and | $N(v_i)$| = | $N(v_i')$| < Δ where 1 ≤i≤5 ,i≠ 2.Hence Δ +1 colors are needed to color N[v_2].

Since $N(v_i) \cap N(v_i') = \varphi$, N[v_2] and N[v_2'] receives same colors whereasv_2 and v_2' receives different colors among the Δ +1 colors.

Hence, EDG(T_1) is Δ +1 conflict-free colorable.

Case (ii) : m≥2 and m is finite

Using case (i) the vertices of V_1 are colored. The vertices in V_2 are colored as follows.

For n= 2,3,4,….m , let l_n= {3n-4,3n,3n+1,3n+2} andS_n = {3n-1}. Now

$$N(v_i) \cap N(v_j) = \begin{cases} \{v_k'\} & \text{if } i,j \in l_n, i \neq j \text{ and } k \in S_n \\ \phi & \text{otherwise} \end{cases}$$

$$N(v_i') \cap N(v_j') = \begin{cases} \{v_k\} & \text{if } i,j \in l_n, i \neq j \text{ and } k \in S_n \\ \phi & \text{otherwise} \end{cases}$$

Also N(v_s) ∩ $N(v_t')$ = $\varphi$for 1 ≤ s,t ≤ 3m+2.

Thus the colors used to color these vertices in V_1 is enough to color the vertices of V_2.

Hence EDG(T_m) is Δ+1 conflict-free colorable.

4. Conclusion

In this paper, we proved the existence of conflict-free coloring in EDG(T_m) and the induced subgraph of each of the color class is a totally disconnected graph. Also we proved that the conflict-free chromatic number of EDG(T_m) is Δ + 1.

REFERENCES

1. A.Dey and A.Pal, Fuzzy graph coloring technique to classify the accidental zone of a traffic control, *Annals of Pure and Applied Mathematics*, 3(2) (2013) 169-178.

2. S.Chandra Kumar and T.Nicholas, b-coloring in square of cartesian product of two cycles, *Annals of Pure and Applied Mathematics*, 1(2) (2012) 131-137.
3. G.Even, Z.Lotker, D.Ron and S.Smorodinsky, Conflict-free colorings of simple geometric regions with applications to frequency assignment in cellular networks, *SIAM Journal on Computing*,33(1) (2003)94-136.
4. R.Gleboy, T.Szabo and G.Tardos, Conflict free coloring of graphs, *Combinatorics, Probability and Computing*, 23(3) (2014) 434 -438.
5. J.Pach and G.Tardos, Conflict-free colorings of graphs and Hypergraphs, Combinatorics, *Probability and Computing*, 18 (2009) 819-834.
6. S.Smorodinsky, Conflict –free coloring and its applications,to appear in *Geometry - Intuitive, Discrete and Convex*,(I.Barany, K.J.Boroczky, G.Fejes Toth, and J. Pach, eds.) Bolyai Society Mathematical Studies, Springer. Preprint available on http://arxiv.org/pdf/1005.3616.pdf.
7. K.Thirusangu, B.Selvam and P.P.Ulaganathan, Cordial labelling in extended duplicate twig graphs, *Int. J. Computer Mathematical Sci. Appl.*, 4(3-4) (2010) 319-328.
8. N.Khan, M.Pal and A.Pal, *L*(0,1)-labelling of cactus graphs, *Communications and Network*, 4 (2012) 18-29.
9. N.Khan and M.Pal, Cordial labelling of cactus graphs, *Advanced Modeling and Optimization*, 15 (2013) 85-101.
10. N.Khan and M.Pal, Adjacent vertex distinguishing edge colouring of cactus graphs, *Inter. J. Engineering and Innovative Technology*, 4(3) (2013) 62-71.
11. S.Paul, M.Pal and A.Pal, L(2,1)-Labeling of Permutation and Bipartite Permutation Graphs, *Mathematics in Computer Science*, DOI 10.1007/s11786-014-0180-2.
12. S.Paul, M.Pal and A.Pal, L(2,1)-labeling of interval graphs, *J. Appl. Math. Comput.*, DOI 10.1007/s12190-014-0846-6.
13. M.Pal and G.P.Bhattacharjee, An optimal parallel algorithm to color an interval graph, *Parallel Processing Letters*, 6 (4) (1996) 439-449.